Multiple UPFCs Mathematical Model Enhancing Multi-Machine Power System Control

G. M. Ali, S. A. Al-Mawsawi

***Abstract*—This paper provides a general dynamic mathematical model of Multiple Unified Power Flow Controllers (UPFC) development; those installed into any selected lines of a Multi-Machine power system network. The development will be based on two UPFCs installed into the New England Power System (NEPS) grid. This mathematical development applicable on any Multi-Machine network based on the required number of UPFCs to achieve power system control enhancement.**

***Index Terms*—Multiple UPFC, Mathematical Model, New England Power System (NEPS), Multi-Machine**

I. Introduction

THE Electric Power Research Institute (EPRI) has initiated a number of research projects devoted to the development of Flexible AC Transmission Systems (FACTS) in which the power flow is dynamically controlled by high voltage dc and related thyristors developments controllers to enhance the controllability and increase power transfer capability, which are designed to overcome the limitation of conventional power flow controllers [1],[2],[3]. Indeed, the Task Force 3 of the IEEE's FACTS Working Group of the DC and FACTS Subcommittee defined the '*Flexible AC Transmission Systems (FACTS)*' as an alternating current transmission systems incorporating power electronic-based and other static controllers to enhance controllability and increase power transfer capability [4]. In 1991, Gyugyi introduced a novel FACTS device, which was the UPFC for the first time, based on the description of the basic principles of the power flow control in power systems [5]. It is the mother of FACTS controllers that provides power flow control, voltage support, transient stability improvement and oscillation damping, etc. simultaneously or selectively [1],[6]. Many of researches have been conducted to represent UPFC steady state mathematical modeling [7], [8], [9], [10]. However in reference [11], Niaki and Iravani developed steady state and dynamic mathematical models of the UPFC. In this paper the Niaki and Iravani model will be used and a general mathematical model will be derived to consider Multi-UPFCs which are installed in a Multi-Machine power system network.

G. M. Ali is with Electrical and Electronics Engineering, University of Bahrain, Isa Town, Bahrain (e-mail: ghassan.hassan@ewa.bh).

S. A. Al-Mawsawi is with Electrical and Electronics Engineering, University of Bahrain, Isa Town, Bahrain (e-mail: aalmossawi@uob.edu.bh).

II. UPFC Mathematical Model

A single phase, schematic diagram of the power circuit of a UPFC is shown in Fig. 1 where the UPFC is installed on a transmission line. The UPFC consists of a Boosting Transformer (BT), an Excitation Transformer (ET), two three-phase Gate-Turn-Off (GTO) based Voltage Source Converters (VSCs), and a DC link capacitors. Furthermore, m_E, m_B, δ_E and δ_B are the amplitude modulation indexes and the phase angle modulation indexes respectively for each VSC, which are the input control signals of the UPFC. The voltage support of the sending-end and shunt reactive power control can be achieved through m_E, whereas the δ_E responsible to control the dc voltage across the dc link capacitor. Furthermore, the line real active and reactive power can be controlled through the series injected voltage magnitude and phase angle (m_B and δ_B) respectively [5].

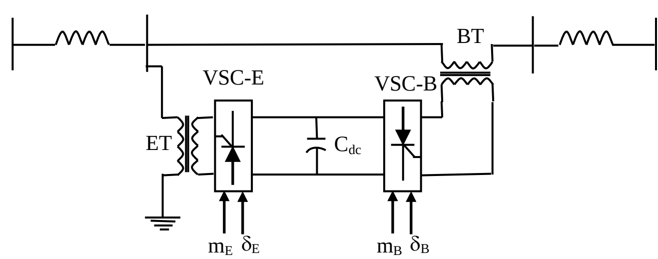

Fig. 1. Schematic diagram of the UPFC power circuit.

In [11] Niaki and Iravani have proposed a comprehensive development procedure and final form of mathematical models of a unified power flow controller UPFC in which pulse width modulation PWM scheme has been used, refer to "(1)", "(2)", "(3)". Consider Fig. 2, which illustrates detailed three-phase circuit diagram of the two transformers BT and ET which are identified by their per phase leakage inductance and resistance, dc link and converters. It is illustrated in Fig. 2 that, each converter leg is composed of a GTO valve and a diode valve in antiparallel connection to permit bi-directional current flow. Based on that work, the dynamic mathematical model of the UPFC is as following:

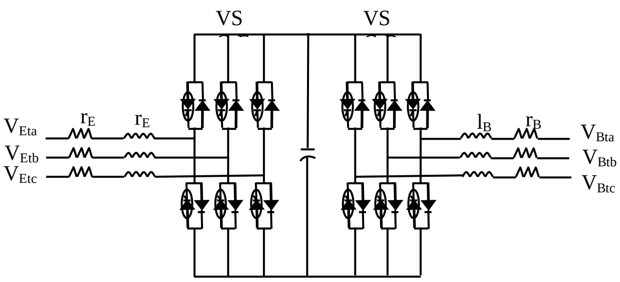

Fig. 2. Detailed three-phase UPFC circuit diagram.

$$\begin{bmatrix} \frac{di_{Ea}}{dt} \\ \frac{di_{Eb}}{dt} \\ \frac{di_{Ec}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{r_E}{l_E} & 0 & 0 \\ 0 & -\frac{r_E}{l_E} & 0 \\ 0 & 0 & -\frac{r_E}{l_E} \end{bmatrix} \begin{bmatrix} i_{Ea} \\ i_{Eb} \\ i_{Ec} \end{bmatrix} - \ldots$$

$$\frac{m_E v_{dc}}{2l_E} \begin{bmatrix} \mathrm{Cos}(\omega t + \delta_E) \\ \mathrm{Cos}(\omega t + \delta_E - 120^\circ) \\ \mathrm{Cos}(\omega t + \delta_E + 120^\circ) \end{bmatrix} + \begin{bmatrix} \frac{1}{l_E} & 0 & 0 \\ 0 & \frac{1}{l_E} & 0 \\ 0 & 0 & \frac{1}{l_E} \end{bmatrix} \begin{bmatrix} v_{Eta} \\ v_{Etb} \\ v_{Etc} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} \frac{di_{Ba}}{dt} \\ \frac{di_{Bb}}{dt} \\ \frac{di_{Bc}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{r_B}{l_B} & 0 & 0 \\ 0 & -\frac{r_B}{l_B} & 0 \\ 0 & 0 & -\frac{r_B}{l_B} \end{bmatrix} \begin{bmatrix} i_{Ba} \\ i_{Bb} \\ i_{Bc} \end{bmatrix} - \ldots$$

$$\frac{m_B v_{dc}}{2l_B} \begin{bmatrix} \mathrm{Cos}(\omega t + \delta_B) \\ \mathrm{Cos}(\omega t + \delta_B - 120^\circ) \\ \mathrm{Cos}(\omega t + \delta_E + 120^\circ) \end{bmatrix} + \begin{bmatrix} \frac{1}{l_B} & 0 & 0 \\ 0 & \frac{1}{l_B} & 0 \\ 0 & 0 & \frac{1}{l_B} \end{bmatrix} \begin{bmatrix} v_{Bta} \\ v_{Btb} \\ v_{Btc} \end{bmatrix} \quad (2)$$

$$\frac{dv_{dc}}{dt} = \frac{m_E}{2C_{dc}} \left[\mathrm{Cos}(\omega t + \delta_E) \quad \mathrm{Cos}(\omega t + \delta_E - 120^\circ) \quad \mathrm{Cos}(\omega t + \delta_E + 120^\circ) \right] \begin{bmatrix} i_{Ea} \\ i_{Eb} \\ i_{Ec} \end{bmatrix}$$
$$- \frac{m_B}{2C_{dc}} \left[\mathrm{Cos}(\omega t + \delta_B) \quad \mathrm{Cos}(\omega t + \delta_B - 120^\circ) \quad \mathrm{Cos}(\omega t + \delta_B + 120^\circ) \right] \begin{bmatrix} i_{Ba} \\ i_{Bb} \\ i_{Bc} \end{bmatrix} \quad (3)$$

By applying Park's transformation for the above dynamic mathematical model of the UPFC, so as to develop time-invariant form of recent equations in two axes (D-Q) rotating reference frame, we have:

$$\begin{bmatrix} \frac{di_{Ed}}{dt} \\ \frac{di_{Eq}}{dt} \\ \frac{di_{E0}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{Ed} \\ i_{Eq} \\ i_{E0} \end{bmatrix} - \frac{m_E v_{dc}}{2l_E} \begin{bmatrix} \mathrm{Cos}\,\delta_E \\ \mathrm{Sin}\,\delta_E \\ 0 \end{bmatrix} + \ldots$$

$$\begin{bmatrix} \frac{1}{l_E} & 0 & 0 \\ 0 & \frac{1}{l_E} & 0 \\ 0 & 0 & \frac{1}{l_E} \end{bmatrix} \begin{bmatrix} v_{Etd} \\ v_{Etq} \\ v_{Et0} \end{bmatrix} + \begin{bmatrix} -\frac{r_E}{l_E} & 0 & 0 \\ 0 & -\frac{r_E}{l_E} & 0 \\ 0 & 0 & -\frac{r_E}{l_E} \end{bmatrix} \begin{bmatrix} i_{Ed} \\ i_{Eq} \\ i_{E0} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} \frac{di_{Bd}}{dt} \\ \frac{di_{Bq}}{dt} \\ \frac{di_{B0}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{Bd} \\ i_{Bq} \\ i_{B0} \end{bmatrix} - \frac{m_B v_{dc}}{2l_B} \begin{bmatrix} \mathrm{Cos}\,\delta_B \\ \mathrm{Sin}\,\delta_B \\ 0 \end{bmatrix} + \ldots$$

$$\begin{bmatrix} \frac{1}{l_B} & 0 & 0 \\ 0 & \frac{1}{l_B} & 0 \\ 0 & 0 & \frac{1}{l_B} \end{bmatrix} \begin{bmatrix} v_{Btd} \\ v_{Btq} \\ v_{Bt0} \end{bmatrix} + \begin{bmatrix} -\frac{r_B}{l_B} & 0 & 0 \\ 0 & -\frac{r_B}{l_B} & 0 \\ 0 & 0 & -\frac{r_B}{l_B} \end{bmatrix} \begin{bmatrix} i_{Bd} \\ i_{Bq} \\ i_{B0} \end{bmatrix} \quad (5)$$

$$\frac{dv_{dc}}{dt} = \frac{3m_E}{4C_{dc}} \left[\mathrm{Cos}\,\delta_E \quad \mathrm{Sin}\,\delta_E \quad 0 \right] \begin{bmatrix} i_{Ed} \\ i_{Eq} \\ i_{E0} \end{bmatrix} - \frac{3m_B}{4C_{dc}} \left[\mathrm{Cos}\,\delta_E \quad \mathrm{Sin}\,\delta_E \quad 0 \right] \begin{bmatrix} i_{Bd} \\ i_{Bq} \\ i_{B0} \end{bmatrix} \quad (6)$$

For simplicity reason the resistance and transient of the BT and ET of the UPFC are ignored. As a result the simplified dynamic mathematical model of the UPFC is achieved as following:

$$\begin{bmatrix} v_{Btd} \\ v_{Btq} \end{bmatrix} = \begin{bmatrix} 0 & -x_B \\ x_B & 0 \end{bmatrix} \begin{bmatrix} i_{Bd} \\ i_{Bq} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} m_B v_{dc}\ \mathrm{Cos}\,\delta_B \\ m_B v_{dc}\ \mathrm{Sin}\,\delta_B \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} v_{Etd} \\ v_{Etq} \end{bmatrix} = \begin{bmatrix} 0 & -x_E \\ x_E & 0 \end{bmatrix} \begin{bmatrix} i_{Ed} \\ i_{Eq} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} m_E v_{dc}\ \mathrm{Cos}\,\delta_E \\ m_E v_{dc}\ \mathrm{Sin}\,\delta_E \end{bmatrix} \quad (8)$$

$$\frac{dv_{dc}}{dt} = \frac{3m_E}{4C_{dc}} \left[\mathrm{Cos}\,\delta_E \quad \mathrm{Sin}\,\delta_E \right] \begin{bmatrix} i_{Ed} \\ i_{Eq} \end{bmatrix} - \frac{3m_B}{4C_{dc}} \left[\mathrm{Cos}\,\delta_B \quad \mathrm{Sin}\,\delta_B \right] \begin{bmatrix} i_{Bd} \\ i_{Bq} \end{bmatrix} \quad (9)$$

By considering Fig. 3 that is a basic mathematical model of the UPFC, where the UPFC is installed between bus-1 and bus-2 in general the following relationship between the bus-voltages and terminal voltages, current flow and voltage sources of the UPFC are derived as following:

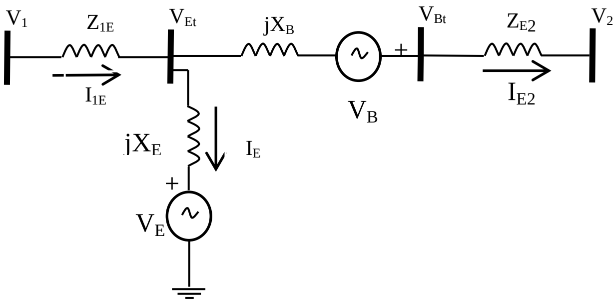

Fig. 3. A basic mathematic mathematical model of the UPFC

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{1E} + jX_E & -jX_E \\ jX_E & -(Z_{E2} + jX_B + jX_E) \end{bmatrix} \begin{bmatrix} I_{1E} \\ I_{E2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} V_E \\ V_B \end{bmatrix} \qquad (10)$$

By solving the system matrix (10) for I_{1E} and I_{E2} as following:

$$\begin{bmatrix} \bar{I}_{1E} \\ \bar{I}_{E2} \end{bmatrix} = \begin{bmatrix} Z_{1E} + jX_E & -jX_E \\ jX_E & -(Z_{E2} + jX_B + jX_E) \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \bar{V}_{E12} \\ \bar{V}_{B12} \end{bmatrix} \right\} \qquad (11)$$

$$\Delta = -Z_{1E}Z_{E2} - jZ_{1E}(X_E + X_B) + X_E(X_B - jZ_{E2}) \qquad (12)$$

$$\begin{bmatrix} \bar{I}_{1E} \\ \bar{I}_{E2} \end{bmatrix} = \frac{1}{\Delta_1} \begin{bmatrix} -(Z_{E2} + jX_B + jX_E)V_1 + jX_E V_2 + (Z_{E2} + jX_B)V_E - jX_E V_B \\ -jX_E V_1 + (Z_{1E} + jX_E)V_2 - Z_{1E}V_E - (Z_{1E} + jX_E)V_B \end{bmatrix} \qquad (13)$$

III. MULTI-MACHINES WITH MULTIPLE UPFCS MATHEMATICAL MODEL

It is decided to install two UPFCs into a multi-machine network such as the New England Power System (NEPS). It consists of 10-machine, 39-bus and 46-line. Consider Fig. 4, which shows the configuration of the NEPS. Indeed, the UPFCs could be installed in any line satisfying the objective of their installation and any practical constrains. So, in this mathematical analysis it will be considered that the UPFC is installed on any line between the concern two buses in general.

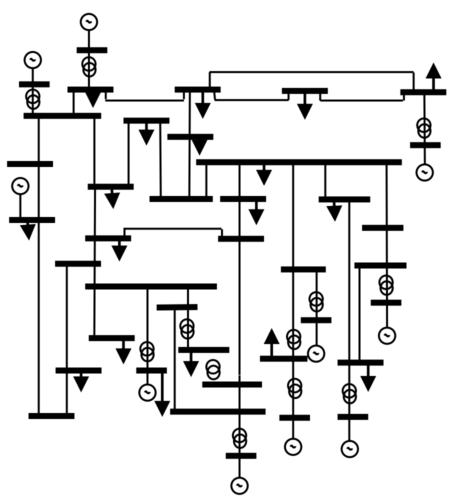

Fig. 4. New England Power System NEPS

The Node Equations of the NEPS will be as following:

$$I = Y\,V \qquad (14)$$

After the installation of two UPFCs the network node equations can be written as:

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \vdots \\ \vdots \\ \bar{I}_{30} \\ \bar{I}_{31} \\ \vdots \\ \bar{I}_{32} \end{bmatrix} = \begin{bmatrix} y_{1,1}+y_{1,2} & 0 & y_{1,3} & y_{1,4} & \cdots & \cdots & \cdots & \cdots & \cdots & y_{1,39} \\ 0 & y_{2,2}+y_{2,1} & y_{2,3} & y_{2,4} & \cdots & \cdots & \cdots & \cdots & \cdots & y_{2,39} \\ y_{3,1} & y_{3,2} & y_{3,3}+y_{3,4} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & y_{3,39} \\ y_{4,1} & y_{4,2} & 0 & y_{4,4}+y_{4,3} & \cdots & \cdots & \cdots & \cdots & \cdots & y_{4,39} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & . & . & . & . & . & . \\ y_{30,1} & y_{30,2} & y_{30,3} & y_{30,4} & . & & & & & y_{30,39} \\ y_{31,1} & y_{31,2} & y_{31,3} & y_{31,4} & & \ddots & & & & y_{31,39} \\ \vdots & \vdots & \vdots & \vdots & & & \ddots & & & \vdots \\ y_{39,1} & y_{39,2} & y_{39,3} & y_{39,4} & & & & \ddots & . & y_{39,39} \end{bmatrix} \begin{bmatrix} \bar{V}_{11} \\ \bar{V}_{21} \\ \bar{V}_{12} \\ \bar{V}_{22} \\ \vdots \\ \vdots \\ \bar{V}_{30} \\ \bar{V}_{31} \\ \vdots \\ \bar{V}_{39} \end{bmatrix} + \begin{bmatrix} I_{1E1} \\ -I_{E21} \\ I_{1E2} \\ -I_{E22} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (15)$$

By substituting for I_{1E1}, I_{E21}, I_{1E2} and I_{E22} in the above system, the following form is achieved:

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \vdots \\ \vdots \\ \bar{I}_{30} \\ \bar{I}_{31} \\ \vdots \\ \bar{I}_{39} \end{bmatrix} = \begin{bmatrix} y_{1,1}+y_{1,2}+\sum x_{11,1} & 0+\sum x_{12,1} & y_{1,3} & y_{1,4} & \cdots & \cdots & \cdots & \cdots & \cdots & y_{1,39} \\ 0+\sum x_{21,1} & y_{2,2}+y_{2,1}+\sum x_{22,1} & y_{2,3} & y_{2,4} & \cdots & \cdots & \cdots & \cdots & \cdots & y_{2,39} \\ y_{3,1} & y_{3,2} & y_{3,3}+y_{3,4}+\sum x_{11,2} & 0+\sum x_{12,2} & \cdots & \cdots & \cdots & \cdots & \cdots & y_{3,39} \\ y_{4,1} & y_{4,2} & 0+\sum x_{21,2} & y_{4,4}+y_{4,3}+\sum x_{22,2} & \cdots & \cdots & \cdots & \cdots & \cdots & y_{4,39} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & . & . & . & . & . & . \\ y_{30,1} & y_{30,2} & y_{30,3} & y_{30,4} & . & & & & & y_{30,39} \\ y_{31,1} & y_{31,2} & y_{31,3} & y_{31,4} & & \ddots & & & & y_{31,39} \\ \vdots & \vdots & \vdots & \vdots & & & \ddots & & & \vdots \\ y_{39,1} & y_{39,2} & y_{39,3} & y_{39,4} & & & & \ddots & . & y_{39,39} \end{bmatrix} \cdots\cdots$$

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \vdots \\ \vdots \\ \bar{V}_{30} \\ \bar{V}_{31} \\ \vdots \\ \bar{V}_{39} \end{bmatrix} + \begin{bmatrix} \frac{Z_{E21}+jX_B}{\Delta_1} & \frac{-jX_{E1}}{\Delta_1} \\ \frac{Z_{1E1}}{\Delta_1} & \frac{Z_{1E1}+jX_{E1}}{\Delta_1} \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{E1} \\ V_{B1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{Z_{E22}+jX_{B2}}{\Delta_2} & \frac{-jX_{E2}}{\Delta_2} \\ \frac{Z_{1E2}}{\Delta_2} & \frac{Z_{1E2}+jX_{E2}}{\Delta_2} \\ 0 & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{E2} \\ V_{B2} \end{bmatrix} \qquad (16)$$

$$\sum x_{11,i} = \frac{-(Z_{E2i} + jX_{Bi} + jX_{Ei})}{\Delta_i},$$

$$\sum x_{12,1} = \frac{jX_{Ei}}{\Delta_i},$$

$$\sum x_{21,i} = \frac{-jX_{Ei}}{\Delta_i},$$

$$\sum x_{22,i} = \frac{Z_{1Ei} + jX_{Ei}}{\Delta_i}$$

Adding the contribution of the internal admittances of the system machines to the machine buses (bus 30 to bus 39), where the machines are connected to get Y_{dd} after the consideration of the internal machine admittances, as following:

$$Y_{dd} = \begin{bmatrix} y_{1,1}+y_{1,2}+\sum x_{11,1} & 0+\sum x_{12,1} & y_{1,3} & y_{1,4} & \cdots & \cdots & \cdots & y_{1,39} \\ 0+\sum x_{21,1} & y_{2,2}+y_{2,1}+\sum x_{22,1} & y_{2,3} & y_{2,4} & \cdots & \cdots & \cdots & y_{2,39} \\ y_{3,1} & y_{3,2} & y_{3,3}+y_{3,4}+\sum x_{11,2} & 0+\sum x_{12,2} & \cdots & \cdots & \cdots & y_{3,39} \\ y_{4,1} & y_{4,2} & 0+\sum x_{21,2} & y_{4,4}+y_{4,3}+\sum x_{22,2} & \cdots & \cdots & \cdots & y'_{4,39} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & . & . & . & . \\ y_{30,1} & y_{30,2} & y_{30,3} & y_{30,4} & . & & y_{30,30}+y_{m30} & y_{30,39} \\ y_{31,1} & y_{31,2} & y_{31,3} & y_{31,4} & & \ddots & & y_{31,39} \\ \vdots & \vdots & \vdots & \vdots & & & \ddots & \vdots \\ y_{39,1} & y_{39,2} & y_{39,3} & y_{39,4} & & & . & y_{39,39}+y_{m39} \end{bmatrix} \qquad (17)$$

$$Y_{machine} = \begin{bmatrix} y_{m30} & 0 & \cdots & \cdots & 0 \\ 0 & y_{m31} & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \cdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & y_{m39} \end{bmatrix} \quad (18)$$

$$Y_b = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & -y_{m30} & 0 & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & -y_{m31} & 0 & \ddots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 & 0 & -y_{m39} \end{bmatrix} \quad (19)$$

So, the following system is the node equation of any multi-machine system installed with multi-UPFCs:

$$\begin{bmatrix} I_g \\ \cdots \\ I_x \end{bmatrix} = \begin{bmatrix} Y_{machine} & \vdots & Y_b \\ \cdots & \cdots & \cdots \\ Y_b' & \vdots & Y_{dd} \end{bmatrix} \begin{bmatrix} E_g \\ \cdots \\ V_{sys} \end{bmatrix} + \begin{bmatrix} 0 \\ \cdots \\ Y_{upfc1} \end{bmatrix} V_{EB1} + \begin{bmatrix} 0 \\ \cdots \\ Y_{upfc2} \end{bmatrix} V_{EB2} + \ldots + \begin{bmatrix} 0 \\ \cdots \\ Y_{upfck} \end{bmatrix} V_{EBk} \quad (20)$$

IV. CONCLUSION

The mathematical dynamic model for multiple UPFCs installed in a Multi-Machine power network (NEPS) has been presented. Furthermore, the derived model will be implemented to study the dynamic behavior with different types of controllers such as PI-controller and other smart controller by using Matlab program.

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