Performance Analysis For Optimal Bidirectional Amplify-and-Forward Relay Networks

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***Abstract*—This paper presents a new method to analyze the bit error rate performance of Bidirectional Amplify-and-Forward Multiple Relay (BAF-MR) networks. A flat-fading channel is considered for this analysis with the relay selection strategy at the destination to choose the highest Signal-to-Noise Ratio (This paper presents a new method to analyze the bit error rate performance of Bidirectional Amplify-and-Forward Multiple Relay (BAF-MR) networks. A flat-fading channel is considered for this analysis with the relay selection strategy at the destination to choose the highest Signal-to-Noise Ratio (*SNR*) relay among multiple relays. The analytical expression of the *SNR* is derived from the high *SNR* region. Further, a new form of average *SNR* (Av*SNR*) is obtained by using the Arithmetic and Geometric Mean inequality (AGM). The *SNR* is then optimized by balancing the energy efficiency and spectral efficiency. The BAF-MR adopts such optimal *SNR* is termed as an Optimal BAF-MR (OBAF-MR) network. The proposed method focuses on analyzing the asymptotic Bit Error Rate (BER) performance of OBAF-MR networks. The analytical expressions are verified using simulation results.**

***Index Terms*—Bit error rate, Amplify-and-Forward relay, optimal relay network, signal-to-noise ratio.**

I. Introduction

Wireless relaying network has been proposed to provide considerable performance improvement in link reliability, spectral efficiency (SE), Energy Efficiency (EE) and BER performance [1]. Its work is based upon the concept of deploying relay nodes between two communication nodes in order to forward the signal sent by a source node to a destination node. Relay node classification is commonly based on the aspect of the transmission scheme, the main types under this aspect are Amplified-and-Forward (AF) and Decode-and-Forward (DF). This paper focuses on AF type and in particular when it works in a bidirectional condition, which is referred to as Bi-directional Amplify-and-Forward Multiple Relay (BAF-MR) networks.

The main function of the AF relay node is amplifying the dispatched signal from source to destination, but this reduces the Signal-to-Noise Ratio (*SNR*) as a result of amplifying the noise. Thus, noise poses a major problem here that affects the error rate performance of BAF-MR networks. [2]. Another challenge for such network is that it typically uses a selection strategy to choose the best *SNR* of relay branch to enhance EE , SE and BER performance [3]. However this strategy requires the *SNR* to be constantly monitored at each branch, and force the receiver to switch between different branches continually to select the best *SNR*. This makes the Probability Error Rate (PER) estimation in the transmission link of the BAF-MR networks a challenging task.

In this context several studies are proposed to develop PER namely: Bit Error Rate (BER), Block Error Rate (BLER), Packet Error rate (PER), or Symbol Error Rate (SER) [4]. One of the early methods proposed by [5] used high *SNR* to quantify the Asymptotic BER (ABER) of wireless transmission over fading channels. It showed that the high *SNR* approximation is useful, specifically for the ABER performance analysis of wireless communications under severe fading conditions.

Asymptotic SER has been investigated by [6] to analyze error probability performance of BAF-MR networks at the low and high *SNR* levels. The high *SNR* region was used to estimate the asymptotic SER expression. Similarly, reference [7] has considered exact SEP performance of BAF-MR network analysis. It calculated SEP in closed form using the Moment Generating Function (MGF) approach for more beneficial analysis [4]. In [8], BLER performance was derived based on the Highest Worse Signal-To-Noise Ratio (HW-*SNR*) of the BAF-MR networks. However, all the mentioned PER studies have used for suboptimal BAF-MR networks.

Optimizing both relay selection and power allocation for BAF-MR networks have presented by [9, 10] to analyze SER and SEP performances respectively. Both studies showed that the network adopted combined optimal relay selection and optimal power is better in performance than the network with optimized power allocation only. Similar results have presented by [11] when the relay selection combining with network coding for BAF-MR networks. Authors in [12] have also showed a similar performance enhancing, but this time for BER performance. It used high *SNR* region for evaluating BER under joining relay selection and power allocation condition.

The above optimal BAF-MR studies focused on the PER performance in term of either optimizing the power allocation or combined relay selection with optimal power allocation. However, the PER estimation for other types of optimal schemes, such as balanced EE and SE as in [13, 14] have not been considered, whereas many wireless applications commonly demand that the probability of error remains below a specific threshold [15]. So , we propose a method to estimate the ABER performance of OBAF-MR satisfying both SE and EE objectives. The ABER is derived based on *SNR* approximation at the high *SNR* region, since this region is typically used for analysing the PER, as discussed in the aforementioned studies.

To make ABER estimation possible for both OBAF-MR and BAF-MR networks, a new form of the average *SNR* (Av*SNR*) is analyzed, using Arithmetic and Geometric Mean inequality (AGM), as the Av*SNR* is a common term for both ABER and EE in the wireless channel [15].

The rest of the paper is organized as follows: Section II outlines the system model; Section III describes the expressions used to calculate the approximation *SNR*; Section IV presents the proposed method for optimal BAF-MR network; Section IV analyzes the proposed ABER method; Section V shows the simulation results. Finally, conclusions and remarks are discussed in Section VI.

II. System Model

Consider two wireless users (S_a and S_b) exchanging information simultaneously in a BAF-MR network using a single antenna over a Rayleigh fading channel. The transmitted signal is modulated using Binary Phase Shift Keying (BPSK). We assume that both users are transmitting to n relays at the same time slot T_{s1}, and the direct link between both nodes is neglected.

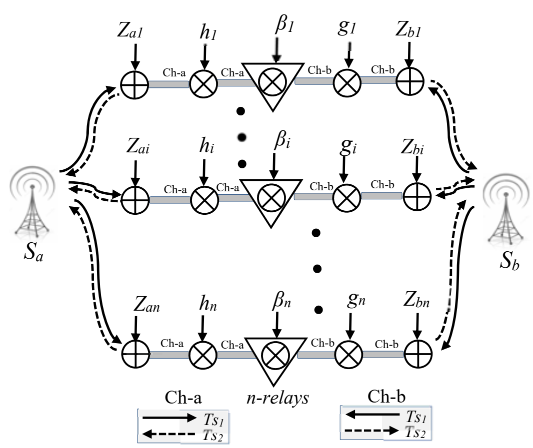

Figure 1. Proposed System Model

The received signal at any relay is given by

$$y_i = \sqrt{p}_a x_a h_i + \sqrt{p}_b x_b g_i + z_{ri} \quad (1)$$

where, y_i is the received signal at the i^{th} relay , P_a, P_b are the signal powers for user S_a and S_b respectively, x_a and x_b are information symbols from each user, h_i is the flat fading channel coefficient between S_a and the relay (r_i), g_i is the flat fading channel coefficient between S_b and R_i, and z_{ri} is the complex noise at the relay Ri .

From (1), the amplification factor for the i^{th} relay can be shown as

$$\beta_i = \sqrt{\frac{p_{ri}}{|h_i|^2 p_a + |gi|^2 p_b + \sigma^2}}, \quad (2)$$

where, β_i is the amplification factor for the i^{th} relay, p_{ri} is the allocated power for R_i, σ^2 is the variance of z_{ri} (we assumed that all noises are to be i.i.d. Gaussian with zero-mean and σ^2 variance).

The signals, once amplified at the relay, are then forwarded to their target users in the second time slot T_{S2} as shown in Figure 1. It is assumed here that the channel conditions are known at the receiving ends. Each user in the network receives amplified signals from all relay branches. However, the one with the highest *SNR* is chosen using a selection scheme.

The received signal for users S_a, S_b can be shown as

$$y_{ai} = y_i h_i \beta_i + z_{ai}, \quad (3)$$

$$y_{bi} = y_i g_i \beta_i + z_{bi}, \quad (4)$$

where, y_{ai} is the received signal for S_a, y_{bi} is the received signal for S_b, z_{ai} and z_{bi} are the Gaussian noises at S_a and S_b respectively.

The exact *SNR* can be calculated for node a using Equations (1) and (3). This gives

$$\Upsilon_{ai} = \frac{p_{ri} p_b \gamma_{hi} \gamma_{gi}}{p_{ri}\gamma_{hi} + p_a \gamma_{hi} + p_b \gamma_{gi} + 1}, \quad (5)$$

where Υ_{ai} is the exact *SNR* at the node S_a, γ_{hi} and γ_{gi} are the instantaneous channel gain-to-noise ratios (CNRs) of first and second hop respectively.

The CNRs can be defined as

$$\gamma_{hi} = |h_i|^2 \sigma^{-2} \quad (6)$$

$$\gamma_{gi} = |g_i|^2 \sigma^{-2}, \quad (7)$$

Both (6) and (7) are independent and identically distributed (i.i.d). Furthermore, $|h_i|^2$ and $|g_i|^2$have the same expectation value equal to one.

Following the similar method to calculate *SNR* at node S_a, the exact *SNR* for node S_b can be shown as

$$\Upsilon_{bi} = \frac{p_{ri} p_a \gamma_{hi} \gamma_{gi}}{p_{ri}\gamma_{hi} + p_a \gamma_{gi} + p_b \gamma_{gi} + 1}, \quad (8)$$

The approximations of Equations (5) and (8) give the approximate *SNR* for users S_a and S_b as

$$\gamma_{ai} = \frac{p_{ri} p_b \gamma_{hi} \gamma_{gi}}{p_{ri}\gamma_{hi} + p_a \gamma_{hi} + p_b \gamma_{gi}} \quad (9)$$

$$\gamma_{bi} = \frac{p_{ri} p_a \gamma_{hi} \gamma_{gi}}{p_a \gamma_{hi} + p_{ri}\gamma_{gi} + p_b \gamma_{gi}} \quad (10)$$

where γ_{ai}, γ_{bi} are the approximate *SNR* for user S_a and $S_b$respectively.

Now, let assume that the node S_a recognizes p_{ri}

receives amplified signals from all relay, we assume it is able to recognize p_{ri} in the following form

$$p_{ri} = \zeta\, p_{a,}, \tag{11}$$

where ζ is the relation between the powers of S_a and r_i

Substituting (11) into (9), the value of γ_{ai} is expressed as function of p_a, p_b and ζ as

$$\gamma_{ai} = 1/\frac{(\zeta+1)}{\zeta\, p_b \gamma_{gi}} + \frac{1}{\zeta\, p_a\, \gamma_{hi}} \tag{12}$$

By applying the formula of AGM to (12), the average value is obtained as

$$\bar{\gamma}_{ai} = \frac{\zeta}{2\sqrt{\zeta+1}}\sqrt{\gamma_{\text{fii}}\, \gamma_{\text{gi}}}\,, \tag{13}$$

where,γ_{fii} is equal to $p_a\, \gamma_{hi}$ and γ_{gi}= $p_b\, \gamma_{gi}$.

III. Analysing OBAF-MR Networks

This section discusses our proposed method to obtain OBAF-MR by combining instantaneous *EE* and *SE* in a balance scheme. The EE is is recognized as the energy consumed by the network to transmit one bit, as

$$EE\text{= Total data delivered } (\Re_i)/ \text{Total energy consumed } (p_{ti})\ , \tag{14}$$

where $\Re_i$ is the total throughput (bits/sec/Hz) and is the p_{ti} is the total energy consumption [16] .

In BAF-MR networks the value of $\Re_i$ is equal to sum rates of the first and the second hops as $\Re_i = \Re_{ba} + \Re_{ab}$, where$\Re_{ba}$ and $\Re_{ab}$ are the throughputs from users S_b and S_a respectively. The value of $\bar{\Re}_{ba}$ is calculated as

$$\begin{aligned}\Re_{ab} &= E\{0.5\log_2(1+\gamma_{ai})\} \\ &= \frac{1}{2}\int_0^\infty \log_2(1+\gamma_{ai})\, pdf_{\gamma_{ai}}(\gamma_{ai})\, d\gamma_{ai}\end{aligned} \tag{15}$$

where 0.5 number is due to the transmission of data during two time slots, $E\{.\}$ is the expectation value and $pdf_{\gamma_{ai}}$ is the total PDF of the Rayleigh fading channels.

Expression (15) can be evaluated by applying Jensen's inequality [17]. This results in

$$\forall\gamma_{\text{fii}}, \gamma_{\text{gi}}:\ \Re_{ab} \le 0.5\log_2(1+\gamma_{ai}) \tag{16}$$

Similar analysis can be also achieved for $\Re_{ba}$. This gives $\Re_{ab} \le 0.5\log_2(1+\gamma_{bi})$

Thus, the total value of $\Re_i$ is obtained from $\Re_{ba}$ and $\Re_{ab}$ as

$$\Re_i \le \frac{1}{2\log_2}\left(\log(1+\gamma_{ai}) + \log(1+\gamma_{bi})\right), \tag{17}$$

The roots from (17) gives $\gamma_{ai} = -\gamma_{bi}/(\bar{\gamma}_{bi}+1)$, $\forall\gamma_{bi} \ne 0$ and $\gamma_{bi} = -\gamma_{ai}/(\bar{\gamma}_{ai}+1)$, $\forall\gamma_{ai} \ne 0$. These roots are substituted in (17) considering properties of logarithm. This gives

$$\bar{\Re}_i \le \frac{1}{2}\log_2(1+\gamma_{ai}) \tag{18}$$

Equation (18) also gives $\phi \le 2^{\overset{*}{\Re}_i} - 1$, where $\gamma_{ai} = \phi$. The expression ϕ is employed to minimize p_a and p_b by following the procedures outlined in [khalil]. This gives

$$p_{a(\phi)} = p_{b(\phi)} = 2\,\frac{\phi}{\zeta}\frac{1}{\sqrt{\bar{\gamma}_{hi}\,\bar{\gamma}_{gi}}} \tag{19}$$

By referring to the equation (14), the term p_{ti}, which includes the power consumption of the amplification circuits and the signal processing, can be expressed as $p_{ti} = \left(\frac{\xi}{2}((\zeta+1)\,p_a + p_b) + (\text{ɲ}\,\Re_i + p_{ci})\right)$, where $(p_{ci} + \text{ɲ}\Re_i)$ is associated with the relay circuit power consumption and includes the static circuit power (p_{ci}) and the dynamic power for a unit data rate ($\text{ɲ}\Re_i$), ξ is a constant related to efficiency of amplifier power.

By using (19) into p_{ti} , we obtain the total energy consumed as function of ϕ ($p_{ti(\phi)}$) as following

$$p_{ti(\phi)} = \left(\frac{\xi}{2}((\zeta+1)\,p_{a(\phi)} + p_{b(\phi)}) + \left(\text{ɲ}\,\Re_{i(\phi)} + p_{ci}\right)\right) \tag{20}$$

where $\Re_{i(\phi)}$ is the data rate in respect to, which can be obtained from (19) and (18).

Substituting $\Re_{i(\phi)}$ and (20) into (14) gives EE as a function of $\phi EE(\phi)$. Then, $EE(\phi)$ derived with respect to ϕ by letting it equal zero, considering that $\Re_{ab}$ is cancelled via subtraction of the back-propagating self-interference [18]. This gives the optimal $\gamma_{ai(\phi)}$ ($\mathring{\gamma}_{ai(\phi)}$) as follows

$$\mathring{\gamma}_{ai(\phi)} = e^{1+w\,(\frac{\Gamma-1}{e})} - 1, \tag{21}$$

where $\mathring{\gamma}_{ai(\phi)}$ is the optimal value of γ_{ai} , e is the base of the natural logarithm, $\Gamma = \frac{(\zeta+1)\,\gamma_{hi} + \gamma_{gi} + (\zeta+2)\,\sqrt{\gamma_{hi}\gamma_{gi}}}{2\gamma_{hi}\gamma_{gi}}$ and $w(.)$ is the real branch of omega function.

Now, equation (12) can be formulated with respect to (21) as

$$\mathring{\gamma}_{ai(\phi)} = 1/\frac{(\zeta+1)}{\zeta\,\mathring{\gamma}_{\text{gi}(\phi)}} + \frac{1}{\zeta\,\mathring{\gamma}_{\text{fii}(\phi)}} \tag{22}$$

where $\mathring{\gamma}_{\text{fii}(\phi)}$ and $\mathring{\gamma}_{\text{gi}(\phi)}$ are the arguments of the function $\mathring{\gamma}_{ai(\phi)}$.

The average value of (22) ($\mathring{\bar{\gamma}}_{ai(\phi)}$) is obtained similar to (13) as

$$\mathring{\bar{\gamma}}_{ai(\phi)} = \frac{\zeta}{2\sqrt{\zeta+1}}\sqrt{\mathring{\gamma}_{\text{fii}(\phi)}\mathring{\gamma}_{\text{gi}(\phi)}} \tag{23}$$

IV. Analysing ABER of OBAF-MR Networks

To analyze the ABER performance of OBAF-MR networks using (21), we can adopt the general BER method of one hop communication system presented by [19]. This can be realized by assuming that two-hop channel is equivalent to a one-hop channel, as the errors at the D node can occur either when the S to r_i transmission is received correctly while the r_i to

D transmission is received in error; or, vice versa. Hence, the error probability at the node D can be equivalent to one-hop as

$$P_e(\mathring{\gamma}_{ai(\phi)}) = \int_0^\infty Q(\sqrt{2\mathring{\gamma}_{ai(\phi)}})\,\frac{e^{-\frac{\mathring{\gamma}_{ai(\phi)}}{\bar{\mathring{\gamma}}_{ai(\phi)}}}}{\bar{\mathring{\gamma}}_{ai(\phi)}}d\mathring{\gamma}_{ai(\phi)}, \tag{24}$$

where $P_e(\mathring{\gamma}_{ai(\phi)})$ is the probability error, $Q(\sqrt{2\mathring{\gamma}_{ai(\phi)}})$ is the Q-function and $e^{-\frac{\mathring{\gamma}_{ai(\phi)}}{\bar{\mathring{\gamma}}_{ai(\phi)}}}/\bar{\mathring{\gamma}}_{ai(\phi)}$ is the probability density function (pdf) of the double-cascade i.i.d Rayleigh fading channels.

Since the *SNR* in i.i.d Rayleigh fading channels follows an exponential distribution, the pdf term in (24) is calculated by using Av*SNR* given by (13). This gives

$$\varphi_{ai}(\mathring{\gamma}_{ai(\phi)}) = \frac{2\sqrt{\zeta+1}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}\,e^{-\frac{2\sqrt{\zeta+1}\mathring{\gamma}_{ai(\phi)}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}} \tag{25}$$

where $\varphi_{ai}(\mathring{\gamma}_{ai(\phi)})$ is the total pdf of two-cascaded Rayleigh fading channels.

Integration of (25) relative to γ_{ai} , yields the Cumulative Distribution Function (CDF) for $\varphi_{ai}(\mathring{\gamma}_{ai(\phi)})$. Thus, the CDF of (25) is then given as

$$\mathcal{F}_{ai}(\mathring{\gamma}_{ai(\phi)}) = 1 - e^{-\frac{2\sqrt{\zeta+1}\mathring{\gamma}_{ai(\phi)}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}}, \tag{26}$$

Using the relationship between the Q-function and (26), the term of $Q(\sqrt{2\mathring{\gamma}_{ai(\phi)}})$ in (24) can be rewritten as $Q(\mathring{\gamma}_{ai(\phi)}) = \sqrt{\mathring{\gamma}_{ai(\phi)}\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}/\sqrt{\zeta+1}}$. By substituting this Q-function, in terms of complementary error function ($erfc$), and (25) into (24), the ABER of OBAF-MR networks is expressed as

$$P_e(\mathring{\gamma}_{ai(\phi)}) = \int_0^\infty e^{-\frac{2\mathring{\gamma}_{ai(\phi)}\sqrt{\zeta+1}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}}\; erfc\left(\sqrt{\mathring{\gamma}_{ai(\phi)}\frac{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}{2\sqrt{\zeta+1}}}\right)d\mathring{\gamma}_{ai(\phi)} \tag{27}$$

Evaluating (27) gives the ABER of OBAF-MR network for a single relay branch (i.e, $i = n = 1$). To extend such analysis for n relays, the order statistics technique presented in [20] is applied to analyze *SNR*

In the proposed BAF-MR network, several signals through independent relay branches are received, and then these signals are combined by one of combining strategies. The selection combiner is adopted in this work to select the highest instantaneous *SNR* among several available relays. Thereby, higher order statistics technique gives

$$\mathring{\gamma}_{ai(\phi)(ai)} = max\{\mathring{\gamma}_{a1(\phi)}, \mathring{\gamma}_{ai(\phi)}, ..., \mathring{\gamma}_{an(\phi)}\}, \tag{28}$$

where $\mathring{\gamma}_{ai(\phi)(ai)}$ is the i_{th} order statistic of *SNR*, $\mathring{\gamma}_{an(\phi)}$ represents the largest order statistic and $\mathring{\gamma}_{a1(\phi)}$ be the smallest one.

The order in (28) requires to obtain CDF of $\mathring{\gamma}_{ai(\phi)(ai)}$. In order to do that, let the CDF of $\mathring{\gamma}_{an(\phi)}$ is denoted as $\mathcal{F}_{(ai)}(\mathring{\gamma}_{an(\phi)})$, and then apply the order statistical analysis. This gives

$$\mathcal{F}_{(ai)}(\mathring{\gamma}_{an(\phi)}) = Prob\left(\mathring{\gamma}_{ai(\phi)(ai)} \le \mathring{\gamma}_{an(\phi)}\right) = \mathcal{F}_{(a1)}(\mathring{\gamma}_{a1(\phi)})\,\mathcal{F}_{(a2)}(\mathring{\gamma}_{a2(\phi)})...\mathcal{F}_{(an)}(\mathring{\gamma}_{an(\phi)}) = \left[\mathcal{F}\mathring{\gamma}_{ai(\phi)}\right]^n. \tag{29}$$

By using 29 for (26), CDF for n relays is obtained as

$$\mathcal{F}_{(ai)}(\mathring{\gamma}_{ai(\phi)}) = \left[1 - e^{-\frac{2\sqrt{\zeta+1}\mathring{\gamma}_{ai(\phi)}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}}\right]^n, \tag{30}$$

By derivative equation (30) relative to $\mathring{\gamma}_{ai(\phi)}$, total pdf is yielded as

$$\varphi_{(ai)}(\mathring{\gamma}_{ai(\phi)}) = n\,\frac{2\sqrt{\zeta+1}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}\,e^{-\frac{2\sqrt{\zeta+1}\mathring{\gamma}_{ai(\phi)}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}} \left(1 - e^{-\frac{2\sqrt{\zeta+1}\mathring{\gamma}_{ai(\phi)}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}}\right)^{n-1}, \tag{31}$$

where $\varphi_{(ai)}(\mathring{\gamma}_{ai(\phi)})$ is total pdf of (30).

Substituting (31) and (30) into (24) considering (28), the ABER expression for n relays is given by

$$P_e(\mathring{\gamma}_{ai(\phi)}) = n\int_0^\infty \frac{2\sqrt{\zeta+1}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}\,e^{-\frac{2\sqrt{\zeta+1}\mathring{\gamma}_{ai(\phi)}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}} \left(1 - e^{-\frac{2\sqrt{\zeta+1}\mathring{\gamma}_{ai(\phi)}}{\zeta\sqrt{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}}\right)^{n} d\mathring{\gamma}_{ai(\phi)}. \tag{32}$$

By evaluating (32), the ABER for n relay is obtained as

$$P_e(\mathring{\gamma}_{ai(\phi)}) = \left(\frac{\zeta}{2}\sqrt{\frac{\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}{\zeta+1}}\left(1 - \sqrt{\frac{1}{1 + \frac{(\zeta+1)4n}{\zeta^2\bar{\mathring{\gamma}}_{\text{fi}i(\phi)}\bar{\mathring{\gamma}}_{\text{g}i(\phi)}}}}\right)\right)^n, \tag{33}$$

where $P_e(\mathring{\gamma}_{ai(\phi)})$ is the ABER of multiple relay OBAF-MR networks.

By using the same equation (33) considering $\gamma_{\text{fi}i}$ and $\gamma_{\text{g}i}$, which are derived in Section 3, the ABER of multiple relay suboptimal BAF-MR network is also obtained.

V. Simulation Results

The results of the proposed methods applied to the system model described in Section II are presented in this section. However, for this analysis we are limiting the number of relays to four (i.e, $n = 4$). Further, we consider two values of ζ as 1 and 2 for validating the results. For every value a comparison between simulated and analytic results are presented.

Figure 2 shows ABER performance of BAF-MR network assuming $\zeta = 1$ (i.e., equal power allocation for all nodes). It can be observed that the ABER performance improves as the number of relays increases in the network. The proposed results approach the simulation results especially at the high *SNR* region. However, there are considerable differences between them in the low *SNR* region. This trend is related to the fact that the analysis is limited to the high *SNR* region.

Figure 3 shows the ABER performance assuming $\zeta = 2$, In this case the relay power value is two times power than S_a and S_b. It can been shown that figure 3 follows a similar trend as that of figure 2. A comparison between figures 2 and 3 shows that the proposed schemes resulted in a better performance in case of $\zeta = 2$ than $\zeta = 1$ in the region of high *SNR*. This is because the higher ζ increases overall received SNR and this leads to lower BER [21].

Both figures 2 and 3 are corresponding to the suboptimal network. The ABER results pertaining to optimal networks are shown in figures 4 and 5. Figures 4 compares between the ABER analytic and simulation results to verify equation (33) assuming $\zeta = 1$. It can be seen from the figure a high match between the analytic and simulation results. It also shows that the ABER performance is enhanced compared to figure 2. This proves that a considerable performance can be achieved by an optimal balance between EE and SE.

Finally, figure 5 shows results of the proposed method when estimating the optimal network assuming $\zeta = 2$. It is clear in this figure that the best ABER performance is obtained at high *SNR*. There is a high degree of similarity between the simulated and proposed analysis in the high *SNR* region.

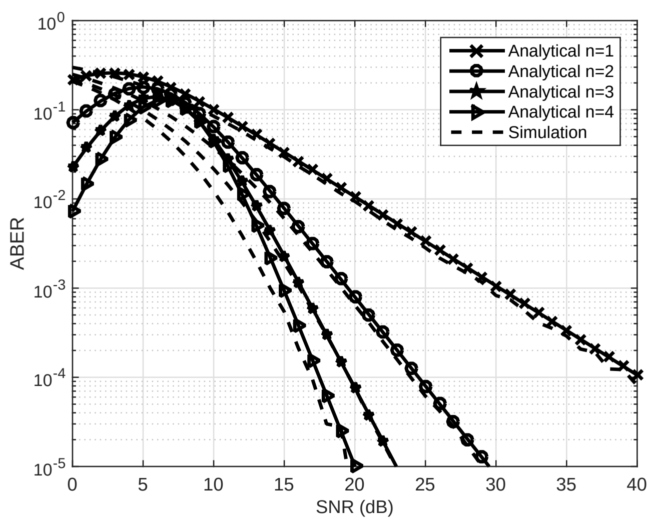

Figure 2. ABER performance assuming $\zeta = 1$

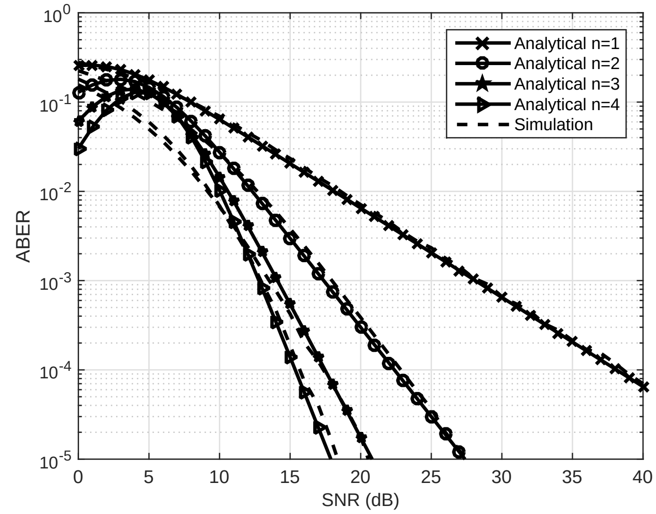

Figure 3. ABER performance assuming $\zeta = 2$

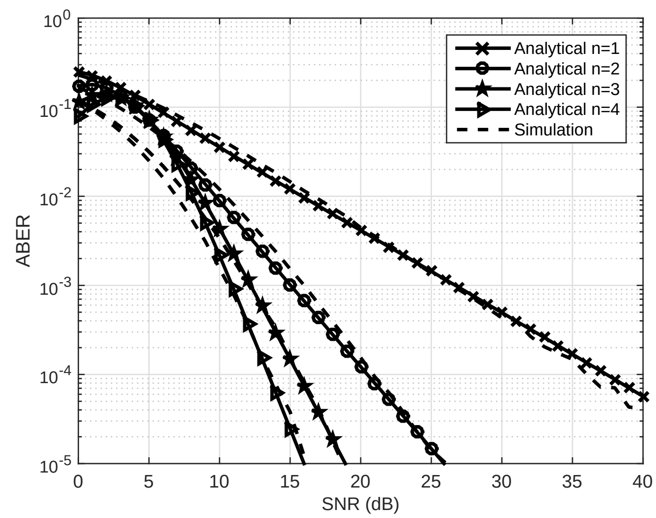

Figure 4. ABER performance assuming $\zeta = 1$ and network is an optimal

VI. Conclusions

A new method is proposed to estimate asymptotic BER performance of OBAF-MR network using flat fading channels and a selection combiner. Analytical expression to calculate the ABER expression is presented using approximated *SNR* and AGM. The AGM provided a beneficial way to calculate an Av*SNR* employed to obtain the ABER for both suboptimal and optimal networks. The optimal network is obtained by balancing energy efficiency and spectral efficiency. The ABER analytic results have been found to align with the simulation results. The method is effective tool to estimate the asymptotic BER of an optimal EE-SE balanced BAF-MR network which shows a lower asymptotic BER than suboptimal networks, a result which has not previously been presented.

References

[1] K. Song, B. Ji, Y. Huang, M. Xiao, and L. Yang, "Performance analysis of antenna selection in two-way relay networks," *Signal Processing, IEEE Transactions on*, vol. 63, no. 10, pp. 2520–2532, May 2015.

[2] L. Tianxi, S. Lingyang, L. Yonghui, H. Qiang, and J. Bingli, "Performance analysis of hybrid relay selection in cooperative wireless systems,"

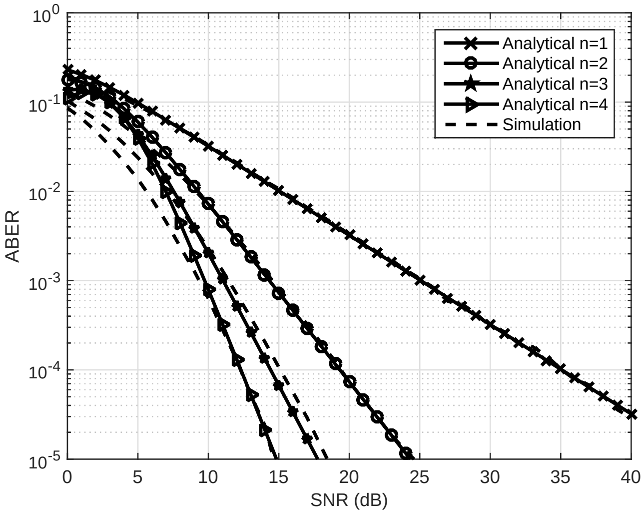

Figure 5. ABER performance assuming $\zeta = 2$ and network is an optimal

Communications, IEEE Transactions on, vol. 60, no. 3, pp. 779–788, 2012.

[3] M. Torabi, D. Haccoun, and J. F. Frigon, "Relay selection in af cooperative systems: An overview," *Vehicular Technology Magazine, IEEE*, vol. 7, no. 4, pp. 104–113, 2012, iD: 1.

[4] M. Dohler and Y. Li, *Introduction*. John Wiley & Sons, Ltd, 2010, pp. 1–41.

[5] Z. Wang and G. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *Communications, IEEE Transactions on*, vol. 51, no. 8, pp. 1389–1398, Aug 2003.

[6] S. S. Ikki and S. Aissa, "Two-way amplify-and-forward relaying with gaussian imperfect channel estimations," *Communications Letters, IEEE*, vol. 16, no. 7, pp. 956–959, 2012.

[7] M. Xia and S. Aissa, "Moments based framework for performance analysis of one-way/two-way csi-assisted af relaying," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 8, pp. 1464–1476, 2012, iD: 1.

[8] S. Atapattu, Y. Jing, H. Jiang, and C. Tellambura, "Relay selection schemes and performance analysis approximations for two-way networks," *Communications, IEEE Transactions on*, vol. 61, no. 3, pp. 987–998, March 2013.

[9] S. Talwar, Y. Jing, and S. ShahbazPanahi, "Joint relay selection and power allocation for two-way relay networks," *Signal Processing Letters, IEEE*, vol. 18, no. 2, pp. 91–94, Feb 2011.

[10] T. P. Do, J. S. Wang, I. Song, and Y. H. Kim, "Joint relay selection and power allocation for two-way relaying with physical layer network coding," *Communications Letters, IEEE*, vol. 17, no. 2, pp. 301–304, February 2013.

[11] Y. Li, R. H. Louie, and B. Vucetic, "Relay selection with network coding in two-way relay channels," *Vehicular Technology, IEEE Transactions on*, vol. 59, no. 9, pp. 4489–4499, Nov 2010.

[12] Z. Fang, F. Liang, L. Li, and L. Jin, "Performance analysis and power allocation for two-way amplify-and-forward relaying with generalized differential modulation," *Vehicular Technology, IEEE Transactions on*, vol. 63, no. 2, pp. 937–942, Feb 2014.

[13] Y. Li and Z. Zheng, "Energy-efficient power allocation for two-hop relay networks," *Electronics Letters*, vol. 50, no. 2, pp. 123–125, 2014.

[14] Y. Li, Z. Zheng, M. Zhao, Y. Chen, and C. Liu, "Energy efficient design for two-way af relay networks," *International Journal of Antennas and Propagation*, vol. 2014, p. 6, 2014.

[15] K. Pahlavan and A. Levesque, *Wireless Information Networks*. Wiley, 2005.

[16] Y. Chen, S. Zhang, S. Xu, and G. Li, "Fundamental trade-offs on green wireless networks," *Communications Magazine, IEEE*, vol. 49, no. 6, pp. 30–37, June 2011.

[17] B. Rao, *A First Course in Probability and Statistics*. World Scientific, 2009.

[18] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *Selected Areas in Communications, IEEE Journal on*, vol. 25, no. 2, pp. 379–389, February 2007.

[19] J. Proakis and M. Salehi, *Digital Communications*. McGraw-Hill Education, 2007.

[20] H.-C. Yang and M.-S. Alouini, "Distributions of order statistics," in *Order Statistics in Wireless Communications*. Cambridge University Press, 2011, pp. 40–71, cambridge Books Online.

[21] M. Dohler and Y. Li, "Transparent relaying techniques," in *Cooperative Communications*. John Wiley & Sons, Ltd, 2010, pp. 141–207.