The Ninth Jordanian International Mechanical Engineering Conference (JIMEC 2018)

Landmark Hotel, Amman – Jordan 16-17 October 2018

MATRIX INVERSION IN FOUR BAR MECHANISM SYNTHESIS USING PATH GENERATION

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Abstract

Path generation is found in many applications of today's industry which can also be the key of inspiration to use many mathematical approaches to synthesize mechanism. Mechanism synthesis using path generation is found in heavy industry such as crane boom or drilling tool in a cnc machine or end effector in robot arm. Many mathematical representations were used to solve the problems in this field such as vectorial methods, exponent methods dimensionless analysis, genetic algorithm, artificial intelligence. This paper presents new mathematical approach to solve the mechanism synthesis using path generation function. The proposed method studies how far the proposed mathematical model (i.e. matrix inversion) ensures the mechanism to go through prescribed poses or try to approximate them with tolerable error.

Keywords: mechanism synthesis, path generation, prescribe points, matrix inversion

1. Introduction

Mechanism synthesis describes the motion of connected links moving relative to each other to achieve certain task through prescribed path. Figure 1 shows four bar mechanism with moving points or pivots ($\mathbf{a}_1$ and $\mathbf{b}_1$) and fixed points or pivots ($\mathbf{a}_0$ and $\mathbf{b}_0$)**.** The mechanism links are described as crank ($\mathbf{a}_0$ $\mathbf{a}_1$), coupler ($\mathbf{a}_1$ $\mathbf{b}_1$) and follower ($\mathbf{b}_0$ $\mathbf{b}_1$).). Figure 1 also shows the motion of mechanism poses in terms of coupler point **p** and coupler displacement angle (α).

Looking into other researchers' work, we can find extensive amount of work in this field Charles W Wampler et al., [1] used numerical polynomial mixed with elimination of variables in nonlinear optimization for mechanism synthesis Jaideep Badduri et al., [2] and N Nariman-Zadeh et al.,[3] used genetic algorithem to syhtesize the mechanism. Computer aided design has great deal in mechanism synthesis, Yuxuan Tong et al., [4] relies on geometric constraints, Hans-Peter Schröcker et al., [5] relies on Euclidean structure,

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Abdulkadar & Deshmukh [6] used software CATIA V5 R19. to check whether the proposed mechanism is providing the required function generation for the given input. Chin Pei Tang [7] used Lagrangian formulation to describes the dynamic equation of a four-bar mechanism. Al-Widyan et al., [8]; Yao and Angeles, [9]; Zhixing et al., [10]; Zhou and Cheung, [11] used nonlinear optimization techniques in solving the coordinates of four bar mechanism.

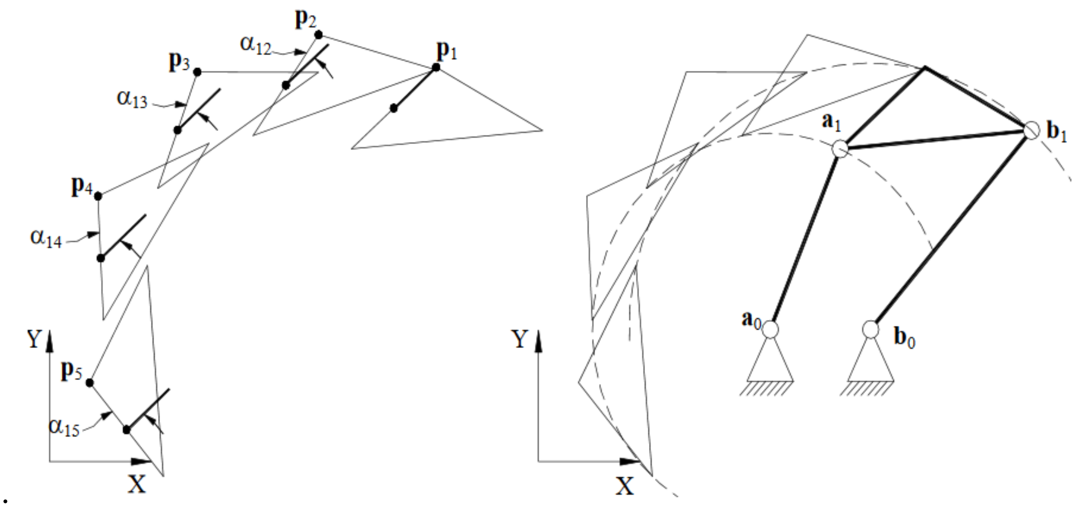

Fig. 1: Proposed mechanism which shows four bar mechanism with prescribed poses [15]

2. Numerical displacement matrices by direct matrix inversion

This work will present the mathematical formulation and derivation of mechanism synthis based on Suh and Radcliff [12]. The basic rotation Matrix equation 1 describes the rotation of any vector fixed in a rigid body. The vector is conveniently described in terms of two points fixed i n the body, a reference point **p** at the tail of the vector, and a point of interest **q** at the head of the vector. For plane rigid body motion (Figure 2), the transformation (i.e. roatation and translation) of points p and q can be written as equation 1

$$\begin{bmatrix} q_x - p_x \\ q_y - p_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} q_{1x} - p_{1x} \\ q_{1y} - p_{1y} \end{bmatrix} \tag{1}$$

Where θ is the rotation of the rigid body with respect to a fixed set of x, y axes that is equivalent to angle α in Figure 1. Equation 1 may be written in the compact form

$$(\mathbf{q} - \mathbf{p}) = [R_\theta](\mathbf{q_1} - \mathbf{p_1}) \tag{2}$$

Typically, the original position $\mathbf{p_1}$ and the final position **p** for the reference point are given along with the rotation angle θ. Equation 2 can then be rearranged in a form suitable for calculation of the coordinates of the new position of point **q** when its first position $\mathbf{q_1}$ is specified.

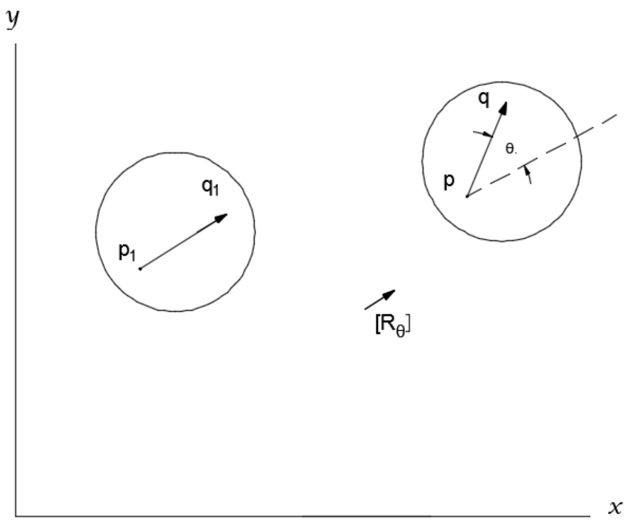

Fig. 2: Plane rigid body displacement.

Solving equation 2 for **q**, we obtain

$$\mathbf{q} = [R_\theta](\mathbf{q_1} - \mathbf{p_1}) + \mathbf{p} \tag{3}$$

Equations 2 and 3 are in a convenient form for carrying out algebraic manipulations and may also serve as the basis for computer subroutines of general usefulness. For plane motion the rotation matrix is 2×2 Matrix.

An alternate form of equation 3 can be found by rearrangement of terms as

$$\mathbf{q} = [R_\theta]\mathbf{q_1} - [R_\theta]\mathbf{p_1} + \mathbf{p} \tag{4}$$

which can be written as a 3×3 matrix equation for plane displacement as

$$\begin{bmatrix} q_x \\ q_x \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_{1i} & -\sin\theta_{1i} & p_{ix} - p_{1x}\cos\theta_{1i} + p_{1y}\sin\theta_{1i} \\ \sin\theta_{1i} & \cos\theta_{1i} & p_{iy} - p_{1x}\sin\theta_{1i} - p_{1y}\cos\theta_{1i} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{1x} \\ q_{1x} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} q_x \\ q_x \\ 1 \end{bmatrix} = \begin{bmatrix} [R_\theta] & (\mathbf{p} - [R_\theta]\mathbf{p_1}) \\ 00 & 1 \end{bmatrix} \begin{bmatrix} q_{1x} \\ q_{1x} \\ 1 \end{bmatrix} \tag{5}$$

Note that a constant z coordinate = 1 has been specified for all points.

$$\begin{bmatrix} \mathbf{q} \\ 1 \end{bmatrix} = [D] \begin{bmatrix} \mathbf{q_1} \\ 1 \end{bmatrix} \tag{6}$$

The 3×3 matrix $[D]$ is the plane displacement matrix. The displacement matrix equation has advantages in repetitive numerical calculations where all matrix elements are defined in terms of the specified displacement of the reference point **p** and the angular displacement of the rigid body. The displacement matrix for pose description is shown in the following equations

Each of the links must satisfy a condition of constant length, where points $\mathbf{a_j}=(a_{jx}, a_{jy})$, $\mathbf{a_0}=(a_{0x}, a_{0y})$, $\mathbf{b_j}=(b_{jx}, b_{jy})$ and $\mathbf{b_0}=(b_{0x}, b_{0y})$ are representative of a typical guiding links $\overline{\mathbf{aa_0}}$ and $\overline{\mathbf{bb_0}}$. These constraint equations become

$$(\mathbf{a_j}-\mathbf{a_0})^T(\mathbf{a_j}-\mathbf{a_0})=(\mathbf{a}_1-\mathbf{a_0})^T(\mathbf{a}_1-\mathbf{a_0}) \qquad j=2,3,\ldots n$$

$$(\mathbf{b_j}-\mathbf{b_0})^T(\mathbf{b_j}-\mathbf{b_0})=(\mathbf{b_1}-\mathbf{b_0})^T(\mathbf{b_1}-\mathbf{b_0}) \qquad j=2,3,\ldots n \tag{7}$$

3. Mathematical Expansion of constant link equations and algorithm

This work considers solving or synthesizing four bar path generation by finding the coordinates of moving and fixed pivots. This synthesis will later determine the link lengths that are responsible for moving the mechanism through or approximately from the prescribed poses. The achived coupler point **q** and coupler displaced angle will then be tested agaisnt prescribed poses.

The maximum number of presribed poses necessary to solve or synthesize the mechanism follows the equations 8 and 9, where n is the number of phases for the mechanism. Unlike the work done by Bhavi & Math [13]. This work will only focus on single phase mechanism (i.e. the mechanism links can not be dismantled in order to move from one pose to another)
The maximum number of rigid body poses can be described as

$$5+3(n-1) \tag{8}$$

Number of unknowns in the crank or the follower links can be described as

$$2+2n \tag{9}$$

For example, for single phase (n=1)four bar mechanism, the maximum number of rigid body poses based on equation 8 is 5, the number of unknows based on equation 9 is 4.

The required unknows are $\mathbf{a_1}=(a_{1x}, a_{1y})$, $\mathbf{a_0}=(a_{0x}, a_{0y})$, $\mathbf{b_1}=(b_{1x}, b_{1y})$ and $\mathbf{b_0}=(b_{0x}, b_{0y})$, so we have two total 8 equations for constant length constraint, with four unknowns a_{1x}, a_{1y}, a_{0x} and a_{0y} for crank and four unknowns b_{1x}, b_1y, b_{0x} and b_{0y} for the follower.

$$[D_{12}]\mathbf{a}_1-\mathbf{a_0})^T([D_{12}]\mathbf{a}_1-\mathbf{a_0})=R_1^{\,2} \tag{10}$$

$$[D_{13}]\mathbf{a}_1-\mathbf{a_0})^T([D_{13}]\mathbf{a}_1-\mathbf{a_0})=R_1^{\,2} \tag{11}$$

$$[D_{14}]\mathbf{a}_1-\mathbf{a_0})^T([D_{14}]\mathbf{a}_1-\mathbf{a_0})=R_1^{\,2} \tag{12}$$

$$[D_{15}]\mathbf{a}_1-\mathbf{a_0})^T([D_{15}]\mathbf{a}_1-\mathbf{a_0})=R_1^{\,2} \tag{13}$$

$$[D_{12}]\mathbf{b}_1-\mathbf{b_0})^T([D_{12}]\mathbf{b}_1-\mathbf{b_0})=R_2^{\,2} \tag{14}$$

$$[D_{13}]\mathbf{b}_1-\mathbf{b_0})^T([D_{13}]\mathbf{b}_1-\mathbf{b_0})=R_2^{\,2} \tag{15}$$

$$[D_{14}]\mathbf{b}_1-\mathbf{b_0})^T([D_{14}]\mathbf{b}_1-\mathbf{b_0})=R_2^{\,2} \tag{16}$$

$$[D_{15}]\mathbf{b}_1-\mathbf{b_0})^T([D_{15}]\mathbf{b}_1-\mathbf{b_0})=R_2^{\,2} \tag{17}$$

4. Example

The mathematical model described in equations 10 to 17 is heavily dependent on matrix approach to study the motion of the four bar mechanism that moves from pose 1 to pose 5 as shown in Figure 5 in Appendix 1. All coordinates are in SI units (cm), path generation program can be programed with prescribed values as a_{0x}=0 and b_{0x}=5 and initial guesses as a_{0y}=0, $\mathbf{a_1}$=(3.9392, 0.6946), b_{0y}=0 and $\mathbf{b_1}$=(6.5773,2.5519). The initial guess for contant length of the the crank and the follower are R1=4 and R2=3, repectively. Initial guess is needed for Newton Raphson technique to solve the line search method through sequential quadratic programing, this is also done by PS Shiakolas et al., [14], where the author concluded that the synthesis is greatly affected by the quality of initial guess.

The prescribed rigid body poses are given in Table 1. Using commercially available software like MathCAD, The mathematical model described in equations 10 to 17 can be programed to synthesize the mechanism. The algorithm output will be as the following.

$\mathbf{a_0}$=(0,0.2154), $\mathbf{a_1}$=(3.8509, 0.7607), $\mathbf{b_0}$=(5,-0.0107) and $\mathbf{b_1}$=(6.5806,2.5487), crank length (R1)=3.8987 and follower length (R2)=3.0062. The crank angle (θ) corrosponds to 10°, 21°, 36°, 55.9997°, 65.9990°. The synthesized mechanism is shown in Figure 3.

The scope of this work is to use path generation to synthesize four bar mechanism, the synthesized mechanism as shown in Table 2 goes approximately through the prescribed coupler points in all poses. The yielded scalar error is the absolute difference in any coupler point between the achieved and prescribed poses as shown in Figure 4. The largest error is 0.025 at pose 5.

Tab.1: Prescribed rigid body poses for planar four-bar mechanism

Pose#	q	α
Pose1	4.6813 , 3.4812	0
Pose2	5.3580 , 4.1747	26.1398
Pose3	5.0508 , 4.6661	35.5006
Pose4	4.2199 , 5.0102	46.3845
Pose5	3.7116 , 4.9375	53.2249

Tab.2: Achieved rigid –body poses for planar four-bar mechanism

Pose#	q	α
Pose1	4.6813 , 3.4812	0
Pose2	5.3399 , 4.1844	14.2739
Pose3	5.0425 , 4.6633	28.49614
Pose4	4.2335 , 5.0084	40.10983
Pose5	3.7333 , 4.9490	45.6743

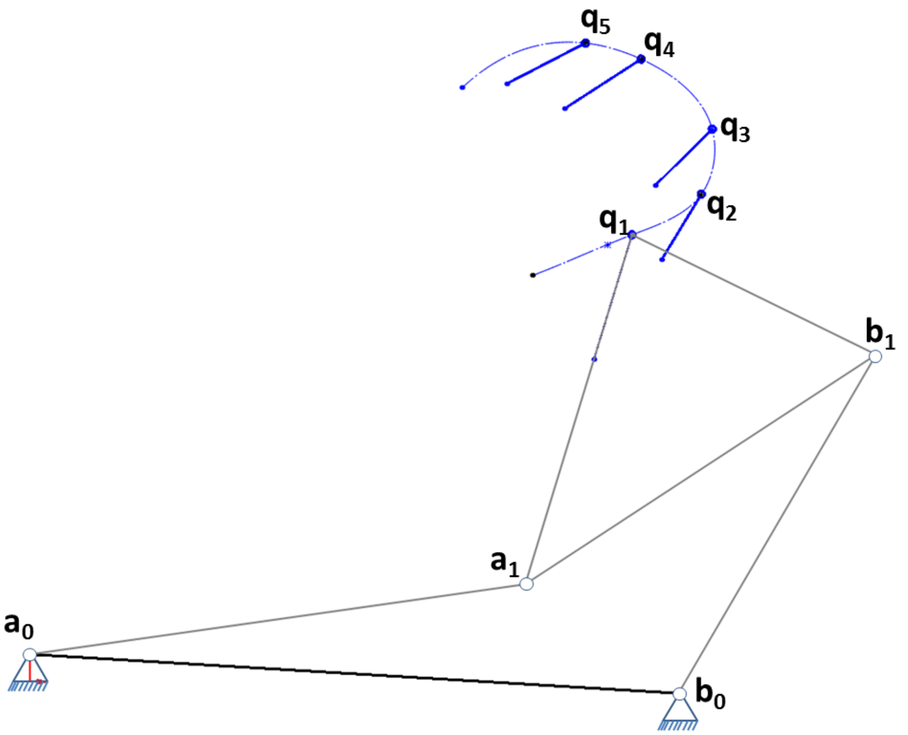

Fig. 3: Four bar mechanism achieved using path generation generated in MathCAD.

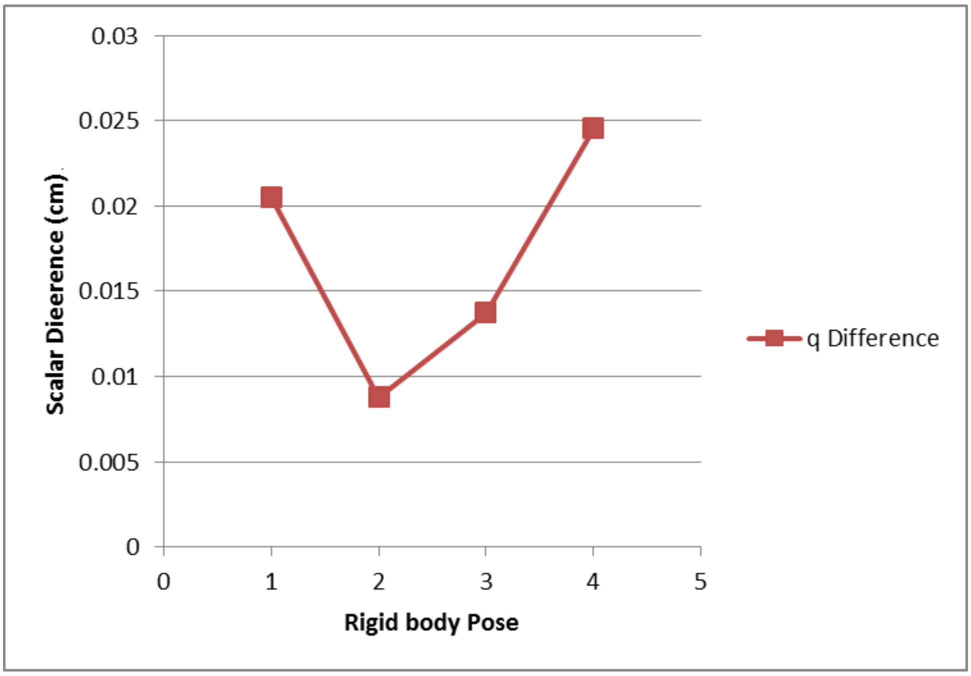

Fig. 4: Scalar difference for coupler point p between the achieved and prescribed poses

5. Discussion

In path generation, the required positions of the coupler are given, equation set 10 to 17 depend on the inversion of displacement matrix which is described by the poses of coupler points. The mathematical model will always produce error. However, this error can be also as a constraint in the modelling condition and can be added to the abovementioned equation set as described in the work of Samer Mutawe [15]. Single phase four bar mechanism is discussed in this work. However, multiphase platform can be formulated and programed as well with few modifications depending on equations 8 and 9, for example if there is adjustable mechanism with two phase (n=2), the maximum number of poses reuired is 8 and the maximum number of unknowns is 6.

6. Conclusions

Matrix inversion method and path generation were used to synthesize four bar mechanism. The achieved mechanism motion passes through prescribed poses with very minimum error of 2.5%. The mathematical model was programmed using MathCAD and mechanism layout and poses were extracted using CAD software such as Solidworks. The proposed method can be used for any four bar application ranging from low to heavy lifting mechanism.

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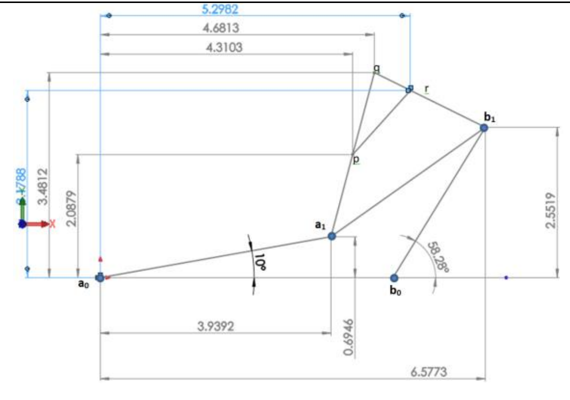
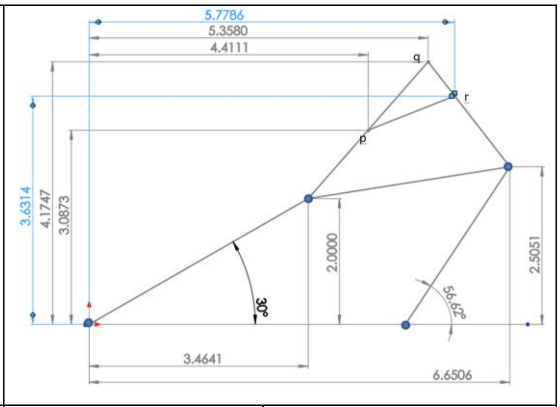
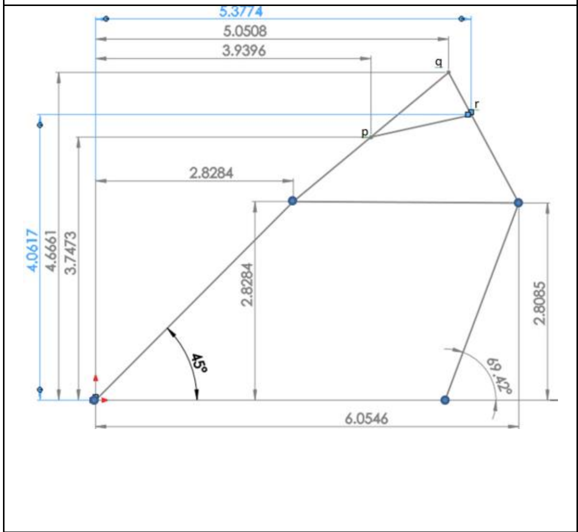
7. Appendix A



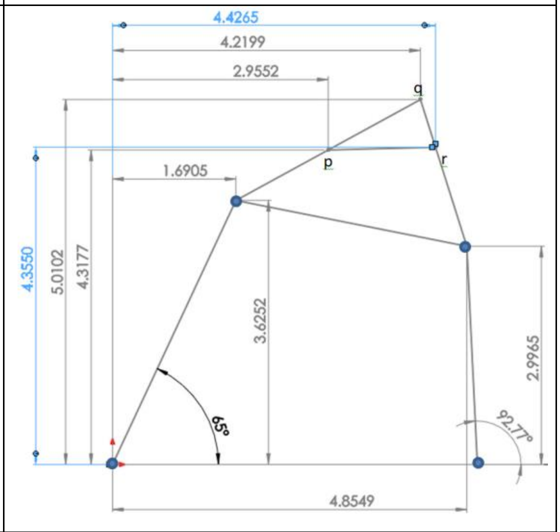
1st pose



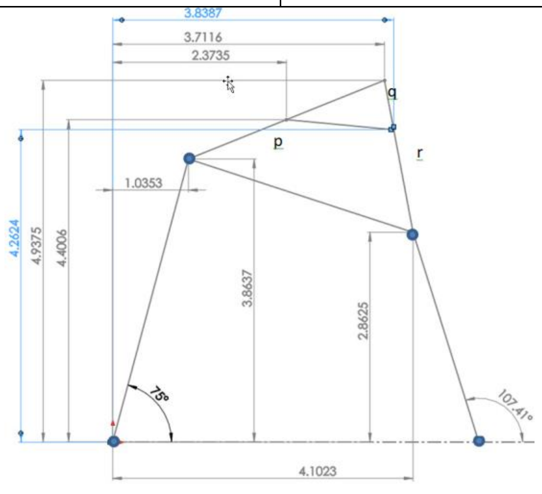
2nd pose



3rd pose



4th pose



5th pose

Fig. 5: four bar mechanism with prescribed poses