Narrow backfill lateral earth pressure

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ABSTRACT

The estimation of narrow backfill lateral earth pressure on yielding and unyielding walls has been and remains a subject of debate among engineers and has been a subject of limited research. Different design methodologies are used by structural engineers with drastically different results.

In this paper an analytical approach to calculate the lateral earth pressure for narrow backfill is presented. This approach is checked against the "Arching Theory" and the results are compared. Furthermore, the Arching theory is extended to include surcharge load on top of the narrow backfill.

It is shown that significant reduction in active or at rest lateral earth pressure acting on yielding and unyielding walls, respectively, is obtained and is justified. This conclusion will have a direct impact on the cost of retaining walls and basement walls construction.

INTRODUCTION

Narrow backfill is a common practice in Jordan and worldwide. The most common case is construction of retaining walls or basement walls a short distance from rock face cut which is usually carried out vertically.

The short distance is usually maintained as work space to allow for formwork, insulation, construction of drains behind the walls and to allow for the protruding width of foundation. Other narrow backfill situations include widening of transportation corridors, within existing right of ways, reduction of rock fall risk and aesthetics.

Lateral earth pressure exerted from a narrow backfill substantially varies and is significantly smaller than the lateral earth pressure exerted by the conventional calculation of apparent earth pressures. The narrow backfill presents a different set of circumstances and parameters that have to be accounted in a different manner for all earth retaining systems.

This paper compares the analytical approaches to calculating the lateral earth pressure from a narrow backfill, compares it to experimental and finite elements data and investigates the affect of using at-rest (ko) vs. active (ka) lateral earth pressure coefficients in determining the distribution and values of the pressure on yielding and unyielding retaining structures with narrow backfill.

BACKGROUND

Several studies have been made to determine the effect of narrow backfill on retaining walls, although none have been widely implemented in practice. The theoretical approaches studied in this work are the Janssen's (1895) arching theory and Barghouthi's two triangle equilibrium solution for which the Coulomb solution is a special case.

Arching Theory

Janssen devised his theory of arching when conducting a series of experiments to determine the lateral and vertical loads on a corn silo. The studies showed that the weight exerted on the bottom of the silo was smaller than the weight of the corn actually placed within the silo's walls. Janssen hypothesized that the friction between the corn and the silo's sidewalls reduced the vertical stress and accordingly developed an equation that predicts the pressure on the silo's sidewalls.

Given the granular nature of the corn within the silo, Janssen's arching theory can be used for granular soils, particularly in retaining walls with narrow backfills. The arching theory shows that the lateral earth pressure is significantly reduced from conventional analysis. This case is illustrated in Figure 1 with the parameters of the problem defined on the same figure. Note that the arching theory as proposed by Janssen only applies to non-yielding walls.

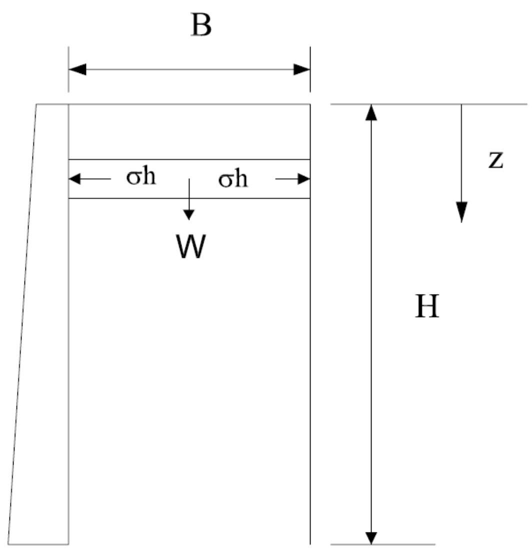

Figure 1 - Illustration of Narrow Backfill

The arching theory predicts the horizontal stresses (σ_h) in Equation 1 below:

$$\sigma_h = \frac{\gamma B}{2tan\delta}\left((1 - e^{-2k\frac{Z}{B}tan\delta})\right) \qquad (1)$$

Where γ = unit weight of soil

B = backfill width

Z = depth from top of wall

H = wall height;

δ = friction angle between the backfill and wall and rock (assumed equal);

k = lateral earth pressure coefficient (assumed Ko for unyielding walls).

$K_o = 1 - \sin \phi`$ (2)

$\phi`$ = effective angle of internal friction for the backfill materials.

Equation 1 can be rearranged to determine the lateral earth pressure coefficient for the limited backfill case as suggested by Kniss et Al (2007) as follows:

$$k' = \frac{1}{2tan\delta} \times \left(\frac{B}{Z}\right) \times \left((1 - e^{-2k\frac{Z}{B}tan\delta})\right) \quad (3)$$

The distribution of lateral earth pressure using Equation 1 is shown in Figure 2 for several cases of backfill width. The equation assumed a unit weight of soil (γ) equal to 1.6 Kg/m^3, a wall height (H) of 9m, a friction angle (δ) of 20^o. Figure 2 shows that the lateral earth pressure with a narrow backfill (B/H = 0.1) is roughly 25% of the calculated at rest pressure and roughly 40% of the exerted forces at the bottom of the backfill.

Take and Valsangkar (2001) conducted an experimental program of centrifuge tests to investigate the arching theory's applicability in soil mechanics. The models were instrumented and calibrated to observe the arching theory with several variations of backfill properties and backfill widths. Results of the experiments verified that arching theory provides a simple and effective tool in estimating the reduction of lateral earth pressure on limited backfill walls.

Figure 3 shows their results for the case with the B/H = 0.107, $\phi = 36^o$ and $\delta = 25^o$.

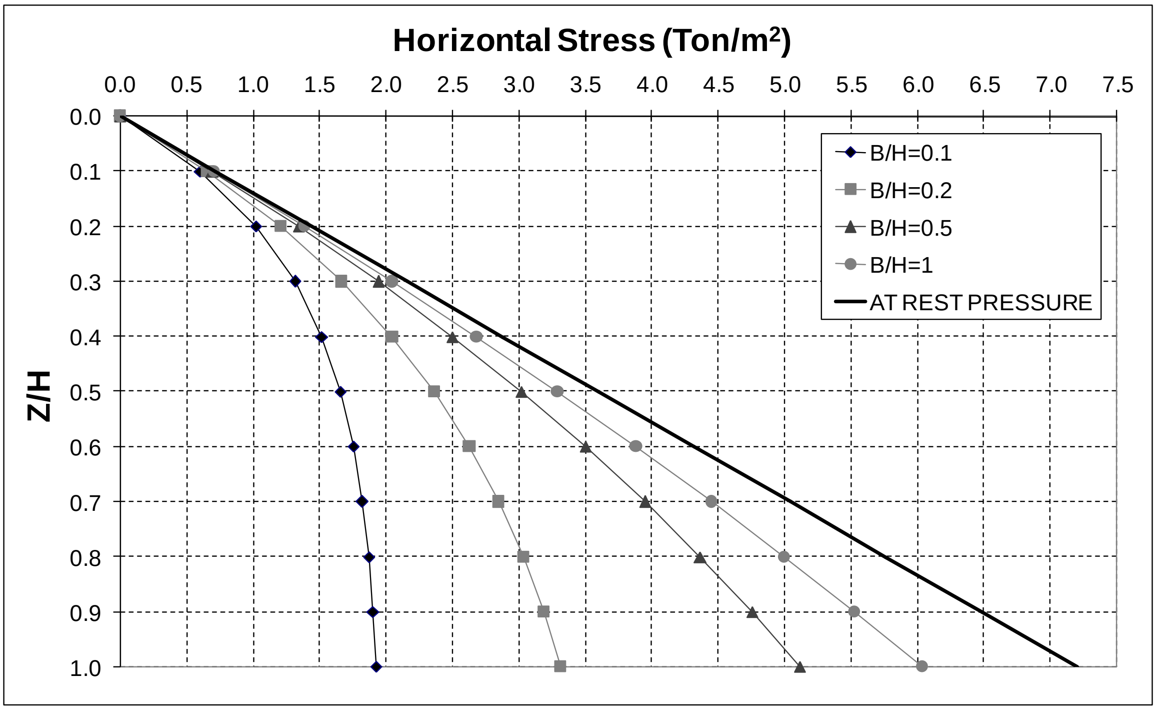

Figure 2 - Distribution of Lateral Earth Pressure with Varying Backfill Widths

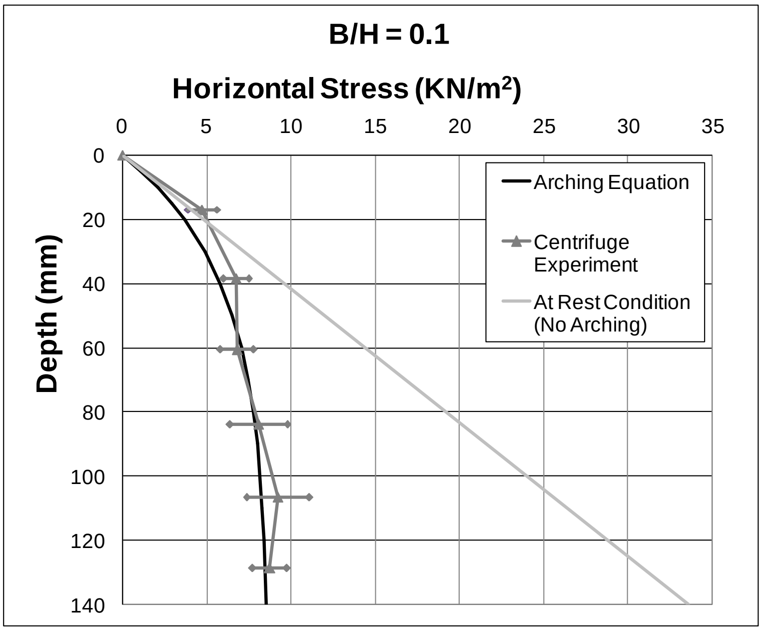

Figure 3 – Centrifgure Experiment Results Verifgying Arching Theory, Reproduced after Take (2001)

Kniss et Al (2007) conducted a series of finite element models using PLAXIS software to investigate whether the analysis matched values calculated from the arching theory and experimental data by Take. The models used the Mohr-Coulomb soil constitutive model with the same parameters as those used in the experiments. Results of the models also showed a good agreement with theory and experimental data for unyielding wall as shown in Figure 4 below.

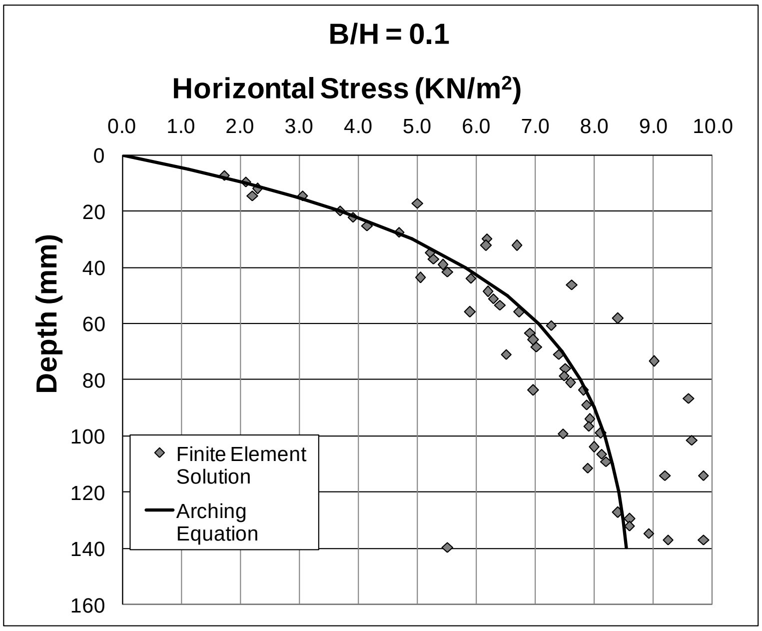

Figure 4 - Finite Element Verification of Arching Theory, Reproduced after Kniss (2007)

EFFECT OF SURCHARGE ON THE ARCHING EQUATION

Although the referenced studies confirm the validity of Janssen's arching theory, none of them consider the effect of surcharge load on arching. Common practice and industry standards typically add a constant surcharge term (surcharge load multiplied by the lateral earth pressures coefficient), which is overestimated in the same manner as the lateral earth pressure from the limited backfill. Using the same arching principals, the friction between the side walls and the backfill is expected to reduce the surcharge loading with an increase height. An equation to calculate the effect of surcharge loading on the arching theory can be derived using the free body diagram in Figure 5.

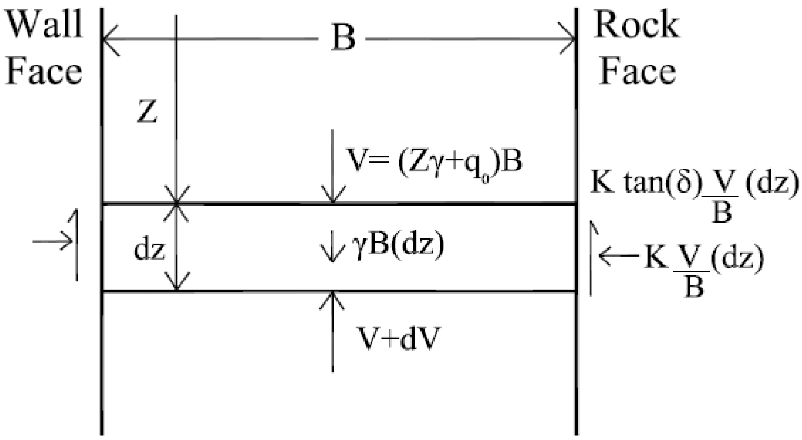

Figure 5 - Free Body Diagram for Arching Equation

The vertical force equilibrium of the horizontal elements of Figure 5 can be calculated as follows:

$$2 \cdot k \cdot tan\delta \cdot \frac{V}{B} \cdot dz = \gamma B \cdot dz - dv \quad (4)$$

Where: V = vertical force at depth Z, which in this case is equal to:

$$V = (\gamma \cdot Z + q_o)B \quad (5)$$

q_o = applied surcharge at the surface

By integrating Equation 4 and introducing the surcharge at the boundary condition where Z=0, the arching affect for both surcharge and lateral earth pressure can be calculated using the following equation:

$$\sigma_{h\ total} = \frac{\gamma B}{2tan\delta}\left((1 - e^{-2k\frac{Z}{B}tan\delta})\right) + k \cdot q_o \cdot e^{-2k\frac{Z}{B}tan\delta} \quad (6)$$

Results of this equation have been plotted for several backfill widths in Figure 6. The plots show a general reduction of surcharge influence on the total lateral load with depth as suggested in the arching theory. Furthermore, the calculated lateral pressures increase with a large backfill width until they match the calculated lateral at-rest triangular distribution.

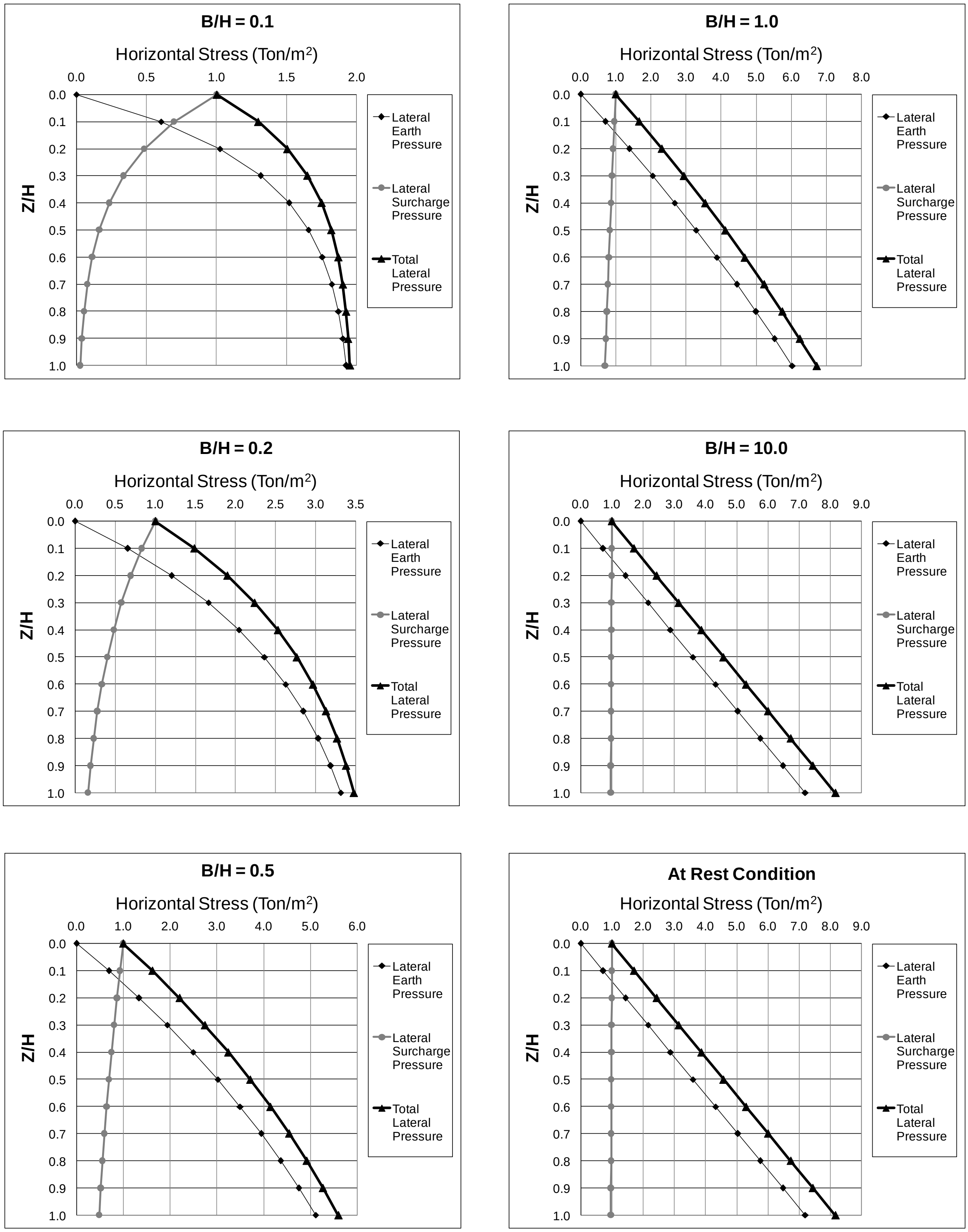

Figure 6 - Arching Effect on Surcharge with Several Variations of Backfill

ARCHING AFFECT ON YIELDING WALLS

Lateral Earth Pressure for Yielding Walls using Wedge Solution

The lateral pressure on yielding walls is estimated using the active earth pressure theory with the coefficient expressed as Ka and calculated by Rankin or Coulomb equations. The coefficient of lateral earth pressure for yielding walls with a narrow backfill can be estimated using the wedge theory shown in Figure 7.

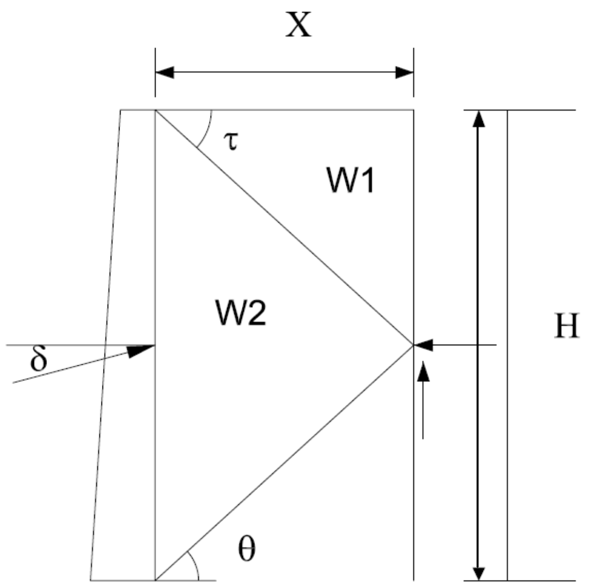

Figure 7 - Two Triangle Lateral Earth Pressure Calculation

By using the two triangle wedge mechanism, Barghouthi (1989) showed that a narrow backfill will indeed reduce the lateral earth pressure on yielding walls, and that the lateral earth coefficient (k_1) at the failure wedge can be calculated by finding the maximum value of the following equation by iterations inθ.

$$K_1 = \left(\frac{X}{H} - \left(\frac{X}{H}\right)^2 tan\theta\right) \cdot \frac{cos\mu\left(\cos(\tau+\varphi)+sin(\tau+\varphi)tan(\theta-\varphi)\right)}{sin(\tau+\varphi+\mu)cos\left(1+tan\delta \cdot tan(\theta-\varphi)\right)} + \frac{X \cdot tan(\theta-\varphi)}{H \cdot cos\delta \cdot \left(1+tan\delta \cdot tan(\theta-\varphi)\right)} \quad (7)$$

Where X = backfill width

H = depth from top of wall

θ = angle of wedge development (see Figure 5)

μ = angle of friction between rock and soil

$$\tau = tan^{-1}\left(\frac{1}{\left(\frac{H}{X}\right)-\tan(\theta)}\right) \quad (8)$$

φ = friction angle of soil

δ = friction angle between the backfill and wall;

The resulting force acting on the wall can then be calculated using Equation 7 as follows:

$$F = \frac{1}{2} K_1 \cdot \gamma \cdot H^2 \quad (9)$$

The distribution of the active earth pressure on yielding walls using Equation 9 is shown in Figure 8 for several cases of backfill width. The figure shows the lateral earth pressure with a limited backfill (B/H = 0.1) is roughly 50% of that calculated active earth pressure at the bottom of the backfill. The calculated pressure is increased as the backfill width increases and the triangular failure wedges are allowed to form. It should be noted that the initial part of all curves is identical, as at the failure wedge is fully formed at the top of the retaining structure.

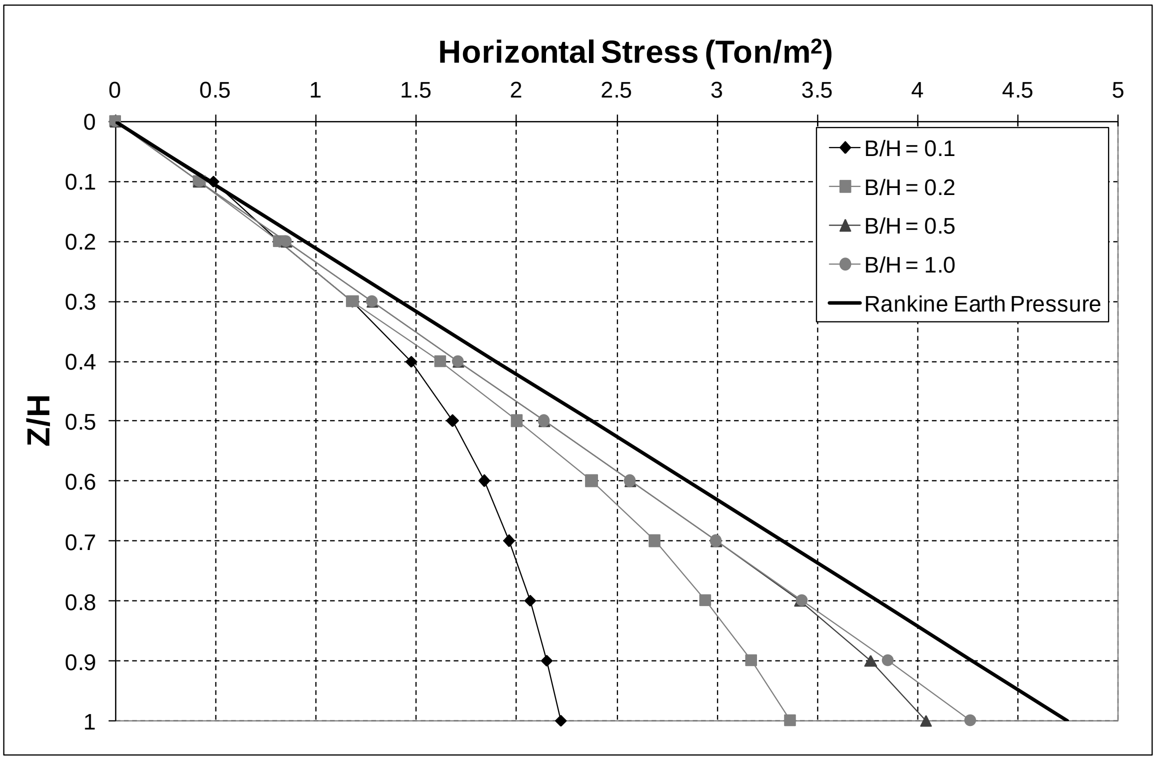

Figure 8 - Distribution of Active Earth Pressure using Wedge Analysis

Arching Theory for Yielding Walls

Janssen's arching theory was initially devised to calculate the lateral earth pressure on unyielding walls. This theory was verified by experimental data as discussed above and shown in Figure 3. However, the principals behind the arching theory and side friction of the backfill also apply to yielding walls with limited deflection at the top of wall. This case is often encountered when foundations are placed on soil that is expected to undergo some form of deformation.

To validate this claim, the lateral earth pressure was calculated as described in Equation 1 using an active lateral earth coefficient (k_A) in Equation (10) and compared to Barghouthi's two triangle solution for yielding walls.

$$k_A = tan^2\left(45 - \frac{\varphi}{2}\right) \quad (10)$$

The resulting horizontal force acting on yielding walls and calculated using the arching equation were obtained by calculating the area under the horizontal stress distribution along the wall lengths. The resulting force at the full wall height was compared to that obtained from Equation 10 using Barghouthi's proposed two triangle solution for several backfill widths as shown in Figure 9.

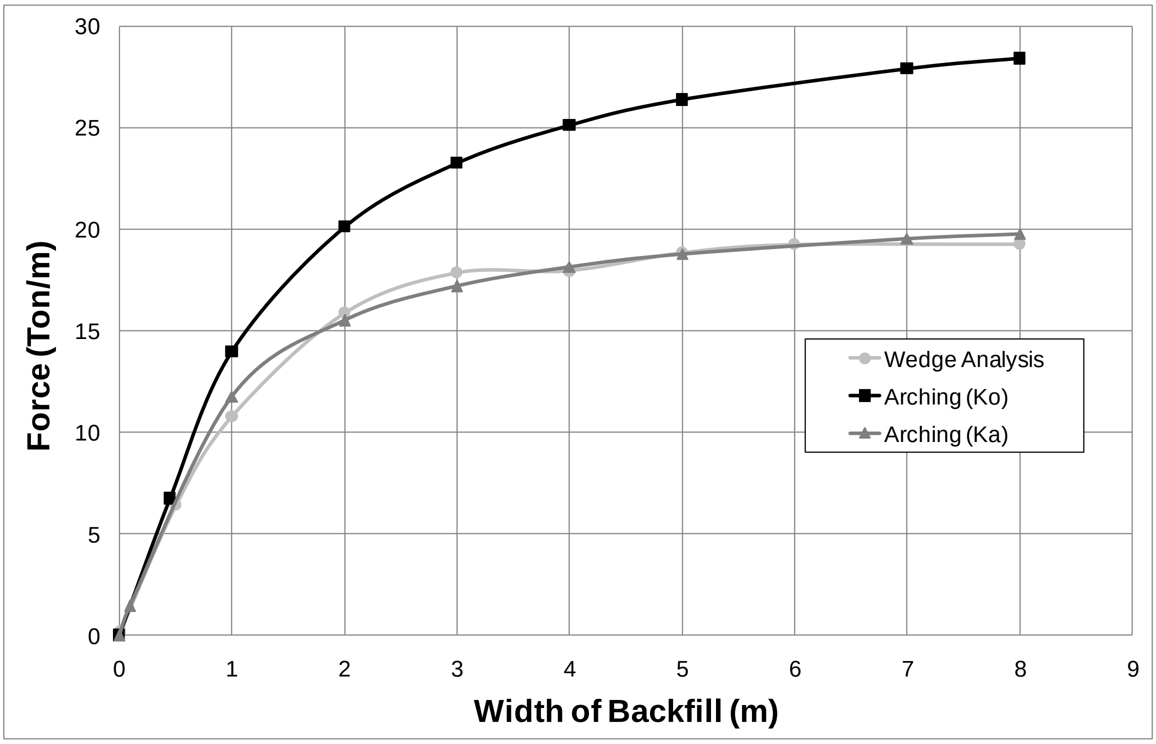

Figure 9 - Comparison of Resultant Forces Acting on Yielding Wall

The calculated forces using the active earth coefficient using the wedge analysis and the arching equation show an almost perfect match, proving that Janssen's can indeed be used of yielding walls.

CONCLUSIONS

Lateral earth pressures on retaining walls with a limited backfill are significantly smaller than earth pressures calculated using apparent earth pressure envelopes. This phenomenon has been explained by Janssen's arching theory and Barghouthi's two triangle wedge analysis.

Janssen's arching equation was expanded to include the surcharge term acting on top of the limited backfill. The surcharge's contribution to the lateral load decreases along the wall height and is significantly reduced at the bottom of the retaining wall. This is primarily due to the vertical friction boundaries located at the interaction of the soil and the wall/rock cut. The equation for calculating the horizontal stresses is derived and presented in this work (Equation 6).

Janssen's theory was also expanded to calculate the horizontal forces exerted on a yielding wall and compared to those calculated using the wedge equilibrium analysis. The results were in strong agreement and verified that the arching equation can be used to determine the load distribution acting on a yielding retaining wall.

Based on the above, the design of retaining walls with narrow backfill can be carried out using reduced forces. For basement walls, the arching theory can be applied using ko conditions. For cantilever retaining walls with narrow backfill where deformations in foundations or deflections in the wall stem are possible, the wedge theory or the arching theory with an active lateral earth coefficient may be used.

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