Studying the effect of a hydrostatic stress/strain reduction factor on damage mechanics of concrete materials

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ABSTRACT

In the Nonlinear Finite Element Analysis (NFEA) of concrete materials, Continuum Damage Mechanics (CDM) provide a powerful framework for the derivation of constitutive models capable of describing the mechanical behavior of such materials. The internal state variables of CDM can be introduced to the elastic analysis of concrete to form elastic-damage models (no inelastic strains), or to the elastic-plastic analysis in order to form coupled/uncoupled elastic-plastic-damage models. Experimental evidence that is well documented in literature shows that concrete's susceptibility to damage and failure is distinguished under deviatoric loading from that corresponding to hydrostatic loading. A reduction factor is usually introduced into a CDM model to reduce the susceptibility of concrete to hydrostatic stresses/strains. In this work, the effect of a hydrostatic stress reduction factor on the performances of two NFEA concrete models will be studied. These (independently published) models did not provide any results showing such effect. One of these two models is an elastic-damage model while the other is an uncoupled elastic-plastic-damage model. Comparisons are carried out between the performances of the two models under tensile and compressive loadings, clearly showing the effect of the reduction factor on the numerically depicted behaviors of concrete materials. In order to have rational comparisons, the hydrostatic stress reduction factor applied to each model is chosen to be a function of the internal state variables common to both models. Therefore, once the two models are calibrated to simulate the experimental behaviors, their corresponding reduction factors are readily available at every increment of the iterative NFEA procedures.

INTRODUCTION

Concrete is a complex highly nonlinear composite material with different mechanical behaviors under different patterns of loading. Furthermore, concrete's material properties are averaged and homogenized rather than accurately determined. These factors, among many others, render the mechanical analysis of such a quasi-brittle material an everlasting challenge.

Several approaches have been applied in the field of numerical modeling of concrete failure, resulting in different categories of constitutive models, such as

CDM models (isotropic and anisotropic), fracture energy models, smeared crack models, and others.

Within the framework of CDM, there are a number of ways to incorporate the damage-related thermodynamic state variables into the NFEA. Restricting this discussion to isotropic damage, some models coupled damage to the elastic analysis of concrete materials, with no consideration of inelastic strains, to form elastic-damage models (Mazars, 1984; Mazars and Pijaudier-Cabot, 1989; Willam et. al., 2001; Tao and Phillips, 2005; Junior and Venturini, 2007; Khan et. al., 2007; and others). Elastic-damage models were accused of being unable to reproduce the unloading slopes during cyclic simulations, unable to capture irreversible strains during plastic flow, and unable to provide an appropriate dilatancy control under multiaxial states of loading. Nevertheless, elastic-damage models are quite noticeable in the literature of engineering mechanics.

Other models introduced damage mechanics variables into the elastic-plastic NFEA of concrete materials, to form either coupled or uncoupled elastic-plastic-damage models. In the uncoupled models, damage is associated with the elastic analysis while the plastic constitutive equations, although present, remained in the effective (undamaged) configuration. Examples of such models are those of Yazdani and Schreyer, 1990; Lee and Fenves, 1998; Shen et. al., 2004; Contrafatto and Cuomo, 2006; Voyiadjis and Taqieddin, 2009; Taqieddin and Voyiadjis, 2009; just to mention a few). These models are superior since they exclude the shortcomings of the elastic-damage models mentioned earlier.

In the coupled elastic-plastic-damage models, the damage variables appear in the elastic as well as the plastic constitutive equations and evolution laws (Luccioni et. al., 1996; Gatuingt and Pijaudier-Cabot, 2002; Salari et. al., 2004; Shao et. al., 2006; Taqieddin et. al., 2011; among others). These models exhibit complex yield criteria, evolution laws, implementation procedures and algorithms, but on the other hand, they are capable of simulating specific material behaviors that are usually ignored in simpler models.

In case of an interest in anisotropic damage mechanics models, and for thoroughness, the reader is referred to the following contributions: Ju, 1989; Yazdani and Schreyer, 1990; Meschke et. al., 1998; Carol et. al., 2001; Voyiadjis et. al., 2008; Voyiadjis et. al., 2009; and the references therein.

For such a quasi-brittle material as concrete, experimental evidence (Resende, 1987) showed that hydrostatic pressure affects the material's yield and failure strength. Some researchers proposed different damage rules to characterize damage in the deivatoric and volumetric modes of response (Resende and Martin, 1984; Papa and Talierco, 1996; just to mention a few), while others applied a hydrostatic stress/strain reduction factor to their constitutive models (Tao and Phillips, 2005; Voyiadjis and Taqieddin, 2009; and others).

The main purpose of this work is to compare the effect of a hydrostatic stress/strain reduction factor on the performances of two isotropic damage models. One of these models is the elastic-damage model proposed by Tao and Phillips, 2005, while the other is the uncoupled elastic-plastic-damage model proposed by Voyiadjis and Taqieddin, 2009. These two models were selected for comparison because both of them defined the hydrostatic stress/strain reduction factor in terms of the same thermodynamic internal state variables. This should not be confused with the fact that the internal state variables themselves are functions of different material properties associated with each model.

The subsequent sections of this paper will introduce, in a brief manner, each of these two models as well as their constitutive equations and material properties relevant to this work. For the full derivations of these models, their numerical integration techniques, and their applications and outcomes, the reader is referred to the already published works of Tao and Phillips, 2005; Voyiadjis and Taqieddin, 2009; and Taqieddin and Voyiadjis, 2009.

Elastic-damage (ED) model

Within the ED model of Tao and Phillips, 2005, the Helmholtz Free Energy (HFE) function is defined in the damaged configuration of the material in terms of the elastic strain equivalence hypothesis (Steinmann et. al., 1994; Lemaitre and Chaboche, 1998; Voyiadjis and Kattan, 2006; and the references therein) and presented as follows:

$$\rho\psi^{e} = (1-\Phi)\rho\bar{\psi}^{e} = \frac{1}{2}\varepsilon_{ij}^{e}E_{ijkl}(\Phi)\varepsilon_{kl}^{e} = \frac{1}{2}(1-\Phi)\varepsilon_{ij}^{e}\bar{E}_{ijkl}\varepsilon_{kl}^{e} = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}^{e} \tag{1}$$

where ψ^{e} is the total free energy density function (per unit volume), entities with an over bar ($\bar{x}$) are those corresponding to the effective (undamaged configurations), ρ is the material's density, $\bar{E}_{ijkl}$ is the fourth-order isotropic elasticity tensor, also known as the undamaged elastic operator, E_{ijkl} is the damaged counterpart of $\bar{E}_{ijkl}$, ε_{ij}^{e} is the elastic strain tensor, σ_{ij} is the Cauchy stress tensor, and Φ is the total damage variable defined as a weighted average function of the damage densities in tension φ^{+} and compression φ^{-}, and expressed as follows:

$$\Phi = \frac{\sum\hat{\sigma}^{+}\varphi^{+} + \sum\left|\hat{\sigma}^{-}\right|\varphi^{-}}{\sum\left|\hat{\sigma}\right|} \tag{2}$$

where $\hat{\sigma}^{+}$ and $\hat{\sigma}^{-}$ are the positive and negative components of the principal stress tensor, respectively, while the term in the denominator is the summation of absolute values of the principal stresses.

Under purely isothermal conditions, the second law of thermodynamics states that any irreversible process within a material's behavior should satisfy the Clausius-Duheim inequality. Applying standard thermodynamic arguments (Coleman and Gurtin, 1967), the following statements are derivable:

$$\sigma_{ij} = \rho\frac{\partial\psi^{e}}{\partial\varepsilon_{ij}^{e}} = (1-\Phi)\bar{\sigma}_{ij} = (1-\Phi)\bar{E}_{ijkl}\varepsilon_{kl} \tag{3}$$

$$Y^{\pm} = -\rho\frac{\partial\psi^{e}}{\partial\varphi^{\pm}} \quad \text{(no mixing)} \tag{4}$$

where $\bar{\sigma}_{ij}$ is the stress tensor in a fictitious undamaged configuration, also known as the effective stress of Kachanov, 1958, Y^{+} and Y^{-} are the damage energy release rates, also known as the thermodynamic conjugate forces associated with tensile φ^+ and compressive φ^- damage, respectively.

In order to distinguish between the different contributions of hydrostatic and deviatoric stress/strain components to damage, and in other words, to reduce the susceptibility of the hydrostatic part to damage, Tao and Phillips, 2005, defined a hydrostatic stress/strain reduction factor χ. They wrote the HFE function, Eq. (1), in terms of the total strain tensor and the hydrostatic mean strain as follows:

$$\rho\psi^e = \frac{1}{2}(1-\Phi)\varepsilon_{ij}^e \bar{E}_{ijkl}\varepsilon_{kl}^e + \frac{1}{2}(1-\chi)\Phi(\frac{1}{9}(\varepsilon_{mm}^e)^2\delta_{ij}\bar{E}_{ijkl}\delta_{kl}) \tag{5}$$

The hydrostatic stress/strain reduction factor χ can take different mathematical forms. Depending on the state of loading, this factor can range from a linear function to a highly nonlinear exponential or power function. Tao and Phillips, 2005, presented the following definition of χ:

$$\chi^{\pm} = 1 - \frac{1}{1 + cY^{\pm}\exp(-dY^{\pm})} \quad \text{(no mixing)} \tag{6}$$

where c and d are two material constants that are calibrated with experimental results, and with units that render the factor χ dimensionless.

The thermodynamic conjugate forces in tension Y^+ and compression Y^- are now readily available using Eqs. (4) and (5):

$$Y^+ = -\rho\frac{\partial\psi^e}{\partial\varphi^+} = \frac{1}{2}\frac{\sum\hat{\sigma}^+}{\sum|\hat{\sigma}|}\left(\varepsilon_{ij}^e\bar{E}_{ijkl}\varepsilon_{kl}^e - \frac{1}{9}(1-\chi^+)(\varepsilon_{mm}^e)^2\delta_{ij}\bar{E}_{ijkl}\delta_{kl}\right) \tag{7}$$

$$Y^- = -\rho\frac{\partial\psi^e}{\partial\varphi^-} = \frac{1}{2}\frac{\sum|\hat{\sigma}^-|}{\sum|\hat{\sigma}|}\left(\varepsilon_{ij}^e\bar{E}_{ijkl}\varepsilon_{kl}^e - \frac{1}{9}(1-\chi^-)(\varepsilon_{mm}^e)^2\delta_{ij}\bar{E}_{ijkl}\delta_{kl}\right) \tag{8}$$

By paying attention to Eq. (6), it becomes obvious that Eqs. (7) and (8) are nonlinear equations that required local iterations during the numerical integration scheme.

Damage initiation under tension or compression is triggered when the thermodynamic conjugate force in tension or compression, respectively, becomes greater than a specified threshold. This is translated into the following two damage criteria:

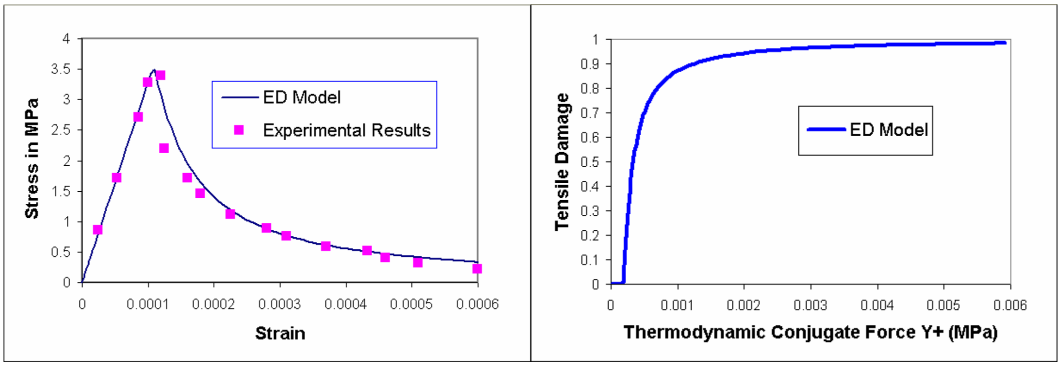
$$g^{\pm} = Y^{\pm} - Y_0^{\pm} - Z^{\pm} \le 0 \quad \text{(no mixing)} \tag{9}$$

where $Y_0^{\pm}$ are the initial damage thresholds in tension (+) and compression (–) which govern the onset of tensile or compressive damage, respectively. Growth or propagation of damage is achieved through hardening/softening parameters $Z^{\pm}$ which were defined by Tao and Phillips, 2005, to take the following form:

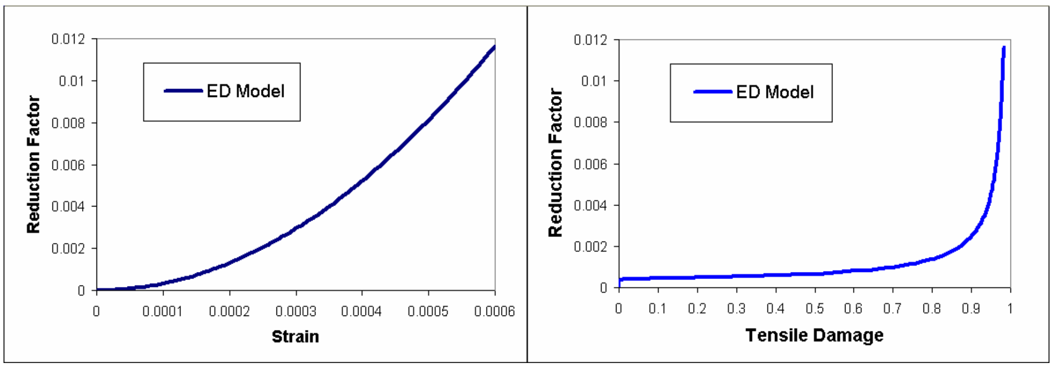
$$Z^{\pm} = \frac{1}{a^{\pm}}\left(\frac{\varphi^{\pm}}{1-\varphi^{\pm}}\right)^{\frac{1}{b^{\pm}}} \quad \text{(no mixing)} \tag{10}$$

in which $a^{\pm}$ and $b^{\pm}$ are four material constants (in tension or compression) to be calibrated by means of uniaxial tensile and compressive experiments of concrete.

This concludes a short introduction to the ED model. The integration procedure of the constitutive equations is thoroughly explained in the work of Tao and

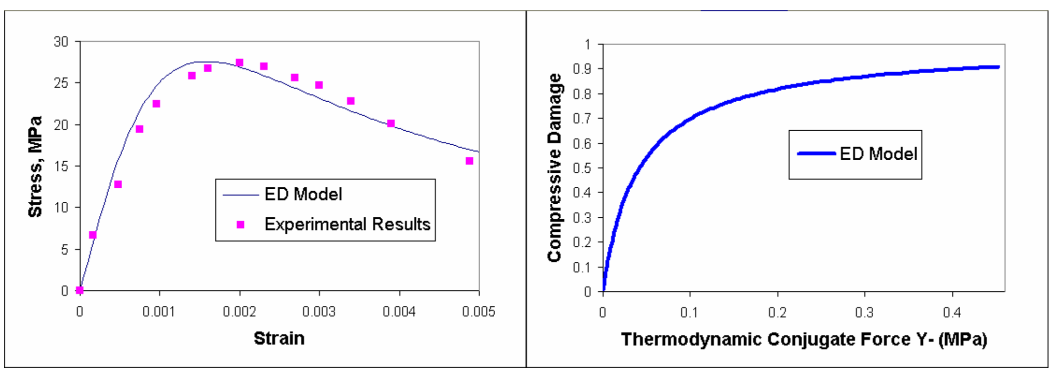


a) Tensile σ_{11} - ε_{11} curve   b) Relation between Y^+ and φ^+

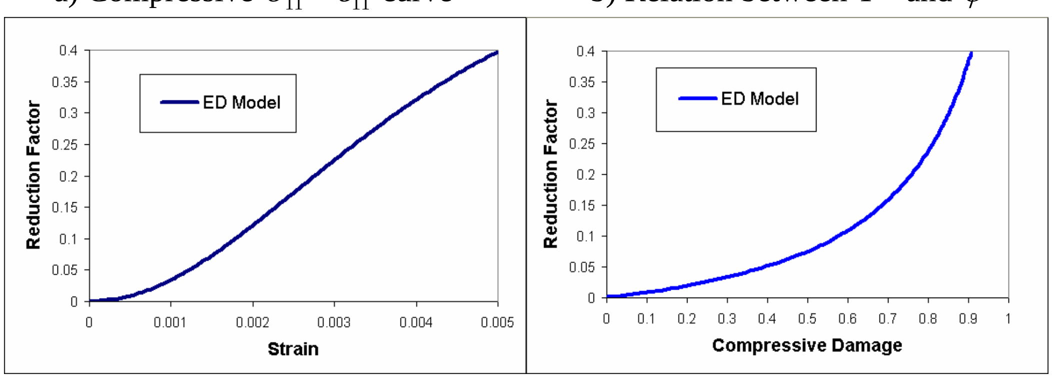


c) Relation between ε_{11} and χ^+ d) Relation between φ^+ and χ^+

Figure 1: Uniaxial tension verification results (ED Model)



a) Compressive σ_{11} - ε_{11} curve   b) Relation between Y^- and φ^-



c) Relation between ε_{11} and χ^- d) Relation between φ^- and χ^-

Figure 2: Uniaxial compression verification results (ED Model)

Phillips, 2005, and will not be discussed here for brevity. In this work, the ED model is implemented into a FORTRAN user defined material subroutine (UMAT) and linked to the NFEA software ABAQUS in order to study the effect of the stress/strain reduction factor χ on the numerical results. The same concrete material properties used by Tao and Phillips, 2005, are adopted here to simulate the required effect. Uniaxial tension and uniaxial compression numerical results are simulated over a single 2D plane-strain verification finite element and presented in Figures (1) and (2).

The experimental results shown in Figure (1a) are the tensile test outcomes of Gopalaratnam and Shah, 1985, while the experimental compressive test results shown in Figure (2a) are those reported by Karsan and Jirsa, 1969.

Uncoupled elastic-plastic-damage (EPD) model

In such a constitutive model, damage mechanics formulations appear in the elastic domain of the material while plasticity remains in the effective (undamaged) stress space (Ju ,1989). Therefore, the strain tensor is additively decomposed into elastic and plastic strain tensors:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \tag{11}$$

and the Cauchy stress tensor is written using the effective stress concept and the equivalent strain hypothesis as follows:

$$\sigma_{ij} = (1-\Phi)\bar{E}_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^{p}) \tag{12}$$

This form of the stress tensor leads to the following incremental constitutive equation ($\dot{x}$ denotes the time derivative of x):

$$\dot{\sigma}_{ij} = (1-\Phi)\bar{E}_{ijkl}\dot{\varepsilon}_{kl}^{e} - \dot{\Phi}\bar{E}_{ijkl}\varepsilon_{kl}^{e} = (1-\Phi)\bar{E}_{ijkl}(\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{p}) - \dot{\Phi}\bar{E}_{ijkl}\varepsilon_{kl}^{e} \tag{13}$$

which requires a special technique known as the elastic-predictor plastic-damage corrector to perform the numerical integration procedure.

In the EPD model of Voyiadjis and Taqieddin, 2009, a multi-hardening pressure-sensitive effective-stress-space plasticity-yield-criterion is introduced in addition to the elastic-damage formulations previously discussed in the ED model (with some adjustments). This plasticity yield criterion is a modification to a criterion first introduced by Lubliner et. at., 1989, and later adopted by Lee and Fenves, 1998; Wu et. al., 2006; and others. It is expressed in terms of the invariants of the effective stress tensor, material hardening functions, and material constants as follows:

$$f = \sqrt{3\bar{J}_2} + \alpha\bar{I}_1 + \beta(\kappa^{\pm})\mathrm{H}(\hat{\bar{\sigma}}_{\max})\hat{\bar{\sigma}}_{\max} - (1-\alpha)c^{-}(\kappa^{-}) = 0 \tag{14}$$

where $\bar{J}_2$ is the second-invariant of the effective deviatoric stress $\bar{s}_{ij} = \bar{\sigma}_{ij} - \bar{\sigma}_{kk}\delta_{ij}/3$, $\bar{I}_1 = \bar{\sigma}_{kk}$ is the first-invariant of the effective stress $\bar{\sigma}_{ij}$, $\kappa^{\pm}$ denote a set of plastic variables chosen to be the equivalent plastic strains in tension and compression (κ^{+}, κ^{-} to be defined in a subsequent paragraph), $\mathrm{H}(\hat{\bar{\sigma}}_{\max})$ is the Heaviside step function (H = 1 for $\hat{\bar{\sigma}}_{\max} > 0$ and H = 0 for $\hat{\bar{\sigma}}_{\max} < 0$), and $\hat{\bar{\sigma}}_{\max}$ is the maximum principal stress. The parameters α and β are defined as a dimensionless constant and a dimensionless function, respectively, and given as follows:

$$\alpha = \frac{(f_{b0}^- / f_0^-) - 1}{2(f_{b0}^- / f_0^-) - 1} \tag{15}$$

$$\beta(\kappa^\pm) = (1-\alpha)\frac{c^-(\kappa^-)}{c^+(\kappa^+)} - (1+\alpha) \tag{16}$$

where f_{b0}^- and f_0^- are the initial equibiaxial and uniaxial compressive yield stresses, respectively, and $c^\pm$ are two internal state variables defined as the plastic hardening functions under uniaxial tension (+) and uniaxial compression (–), respectively. The detailed expressions of $c^\pm$ proposed by Voyiadjis and Taqieddin, 2009, are as follows:

$$c^+(\kappa^+) = f_0^+ + h\kappa^+ \tag{17}$$

$$c^-(\kappa^-) = f_0^- + Q\left[1-\exp(-\omega\kappa^-)\right] \tag{18}$$

where f_0^+ and f_0^- are the uniaxial tensile and compressive yield stresses, respectively, h is a material constant obtained from the uniaxial tensile stress-strain diagram, Q and ω are material constants characterizing the compressive saturated stress and the rate of saturation, respectively.

The equivalent plastic strains in tension and compression (κ^+, κ^-) are defined by the following two expressions:

$$\kappa^\pm = \int_0^t \dot{\kappa}^\pm dt \qquad \text{(no mixing)} \tag{19}$$

and $\dot{\kappa}^+$ and $\dot{\kappa}^-$ are the tensile and compressive equivalent plastic strain rates, respectively, given as:

$$\dot{\kappa}^+ = r(\hat{\bar{\sigma}}_i)\hat{\dot{\varepsilon}}_{\max}^p \tag{20}$$

$$\dot{\kappa}^- = -(1-r(\hat{\bar{\sigma}}_i))\hat{\dot{\varepsilon}}_{\min}^p \tag{21}$$

where $\hat{\dot{\varepsilon}}_{\max}^p$ and $\hat{\dot{\varepsilon}}_{\min}^p$ are the two extreme eigenvalues of the plastic strain rate tensor $\dot{\varepsilon}_{ij}^p$, and $r(\hat{\bar{\sigma}}_i)$ is a dimensionless weight factor $0 \le r(\hat{\bar{\sigma}}_i) \le 1$ defined as:

$$r(\hat{\bar{\sigma}}_i) = \sum_{i=1}^{3}\left\langle \hat{\bar{\sigma}}_i \right\rangle \Big/ \sum_{i=1}^{3}\left|\hat{\bar{\sigma}}_i\right| \tag{22}$$

where $\hat{\bar{\sigma}}_i$ ($i = 1,2,3$) are the effective principal stresses, and the symbol $\langle\ \rangle$ is the Macauley bracket, defined as $\langle x \rangle = \frac{1}{2}(|x| + x)$.

The EPD model of Voyiadjis and Taqieddin, 2009, is consistently derived within the framework of irreversible thermodynamics. The HFE function is defined in terms of a suitable set of elastic and plastic internal state variables and presented as follows:

$$\rho\psi = \rho\psi(\varepsilon_{ij}^e, \kappa^+, \kappa^-, \varphi^+, \varphi^-) = \rho\psi^e(\varepsilon_{ij}^e, \varphi^+, \varphi^-) + \rho\psi^p(\kappa^+, \kappa^-) \tag{23}$$

Applying the internal state variable procedure of Coleman and Gurtin, 1967, followed by the Lagrange minimization procedure (calculus of functions of several variables), the following thermodynamic laws relating the internal state variables to their corresponding conjugate forces are derivable. These laws are lumped here in a single equation for shortness:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}^{p} \frac{\partial F^{p}}{\partial \bar{\sigma}_{ij}} \ , \ \dot{\varphi}^{+} = \dot{\lambda}_{d}^{+} \frac{\partial g^{+}}{\partial Y^{+}} \ , \ \dot{\varphi}^{-} = \dot{\lambda}_{d}^{-} \frac{\partial g^{-}}{\partial Y^{-}} \ , \ \dot{\kappa}^{+} = \dot{\lambda}^{p} \frac{\partial f}{\partial c^{+}} \ , \ \dot{\kappa}^{-} = \dot{\lambda}^{p} \frac{\partial f}{\partial c^{-}} \qquad (24)$$

where $\dot{\lambda}^{p}$ and $\dot{\lambda}_{d}^{\pm}$ are the plasticity and damage Lagrange multipliers, respectively. For the complete derivations, the reader is referred to the work of Voyiadjis and Taqieddin, 2009, and Taqieddin and Voyiadjis, 2009. The term F^{p} is adopted in order to indicate the use of a non-associative plasticity flow rule in the constitutive equations. A non-associated flow rule means that the yield function f and the plastic potential F^{p} do not coincide, and therefore, the direction of the plastic flow is not normal to the yield surface. This is important for realistic modeling of the volumetric expansion (dilatancy) under compression for frictional materials such as concrete. The plastic potential function adopted in this EPD model is a Drucker-Prager type function expressed as follows:

$$F^{p} = \sqrt{3\bar{J}_{2}} + \alpha^{p} \bar{I}_{1} \qquad (25)$$

which facilitates the following derivative:

$$\frac{\partial F^{p}}{\partial \bar{\sigma}_{ij}} = \frac{3}{2} \frac{\bar{s}_{ij}}{\sqrt{3\bar{J}_{2}}} + \alpha^{p} \delta_{ij} \qquad (26)$$

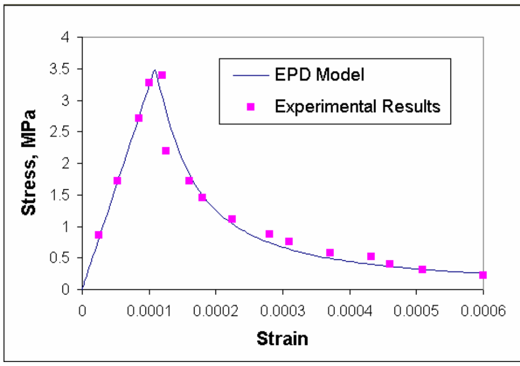
where α^{p} is a parameter chosen to provide proper dilatancy (Lee and Fenves, 1998; and Wu et. al., 2006).

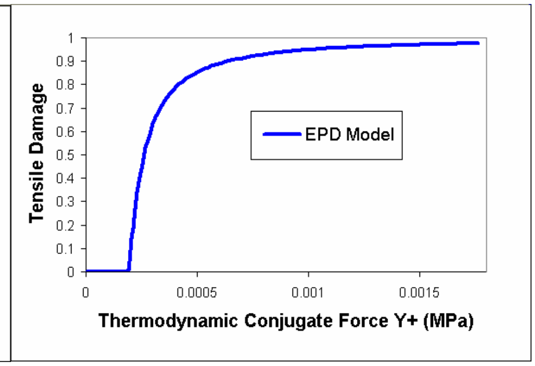
This concludes a very brief introduction to the EPD model of Voyiadjis and Taqieddin, 2009. Many details were overlooked for conciseness, especially those related to the numerical integration procedure. Again, the reader is referred to the work of Voyiadjis and taqieddin, 2009, and Taqieddin and Voyiadjis, 2009, for a comprehensive coverage of all numerical integration aspects.

In this work, the EPD model is also implemented into a UMAT subroutine and linked to ABAQUS in order to study the effect of the stress/strain reduction factor χ on the numerical results. The same concrete material properties used by Voyiadjis and Taqieddin, 2009, are adopted here to simulate the required effect. Uniaxial tension and uniaxial compression numerical results are simulated over the same verification finite element described in the ED model. These results are presented in Figures (3) and (4). The experimental results in these figures are the same as those demonstrated in the ED model.

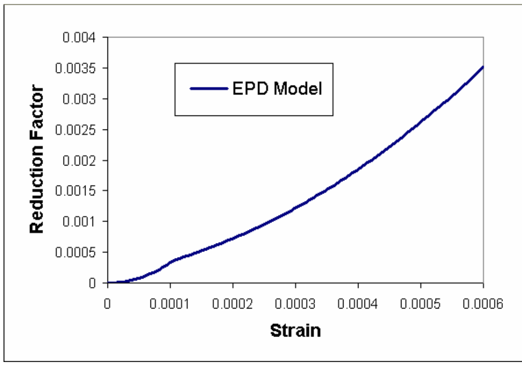
Comparisons and Conclusions

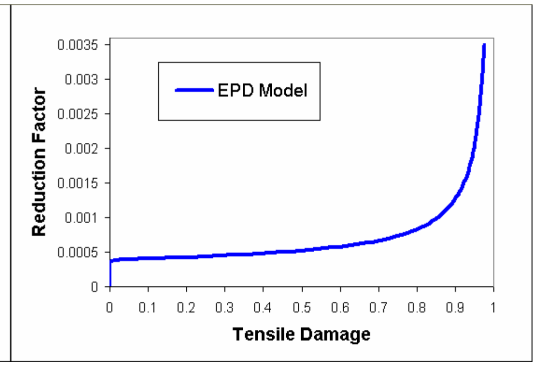
Many conclusions can be drawn based on the presented results; nevertheless, the objective of this work will be the main focus of the comparisons. Figures (5a) and (6a) show the uniaxial stress-strain results of the ED and EPD models under tension and compression, respectively. Figures (5b) and (6b) show the strains plotted against the reduction factors in the tension and the compression verification tests, respectively. In the case of uniaxial tension, and although the two models are similarly capable of reproducing the experimental results, the magnitude of the





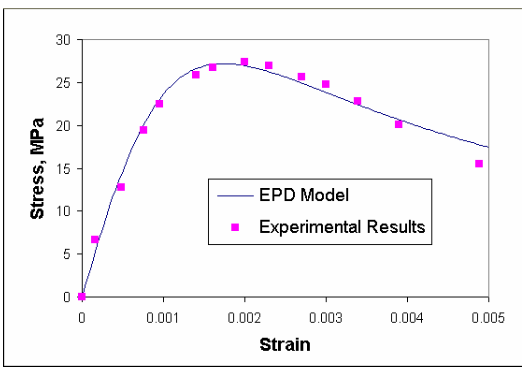
a) Tensile σ_{11} - ε_{11} curve b) Relation between Y^+ and φ^+

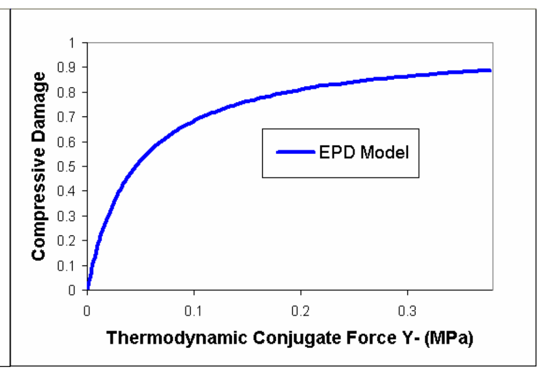




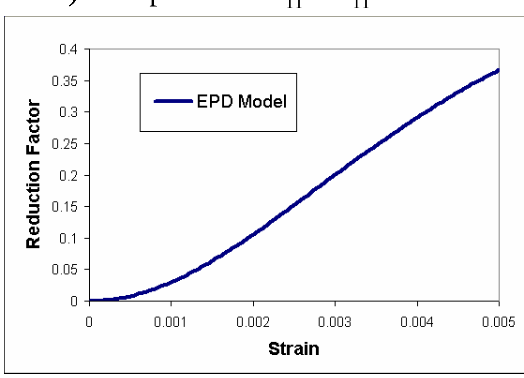
c) Relation between ε_{11} and χ^+ d) Relation between φ^+ and χ^+

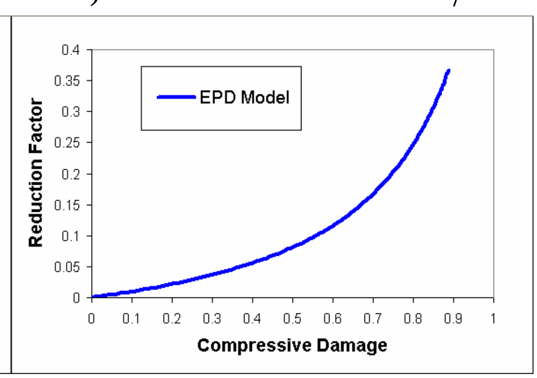
Figure 3: Uniaxial tension verification results (EPD Model)





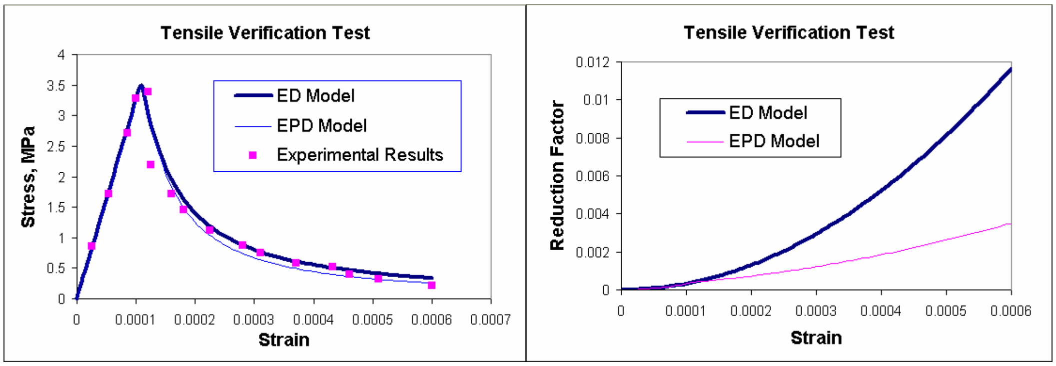
a) Compressive σ_{11} - ε_{11} curve b) Relation between Y^- and φ^-





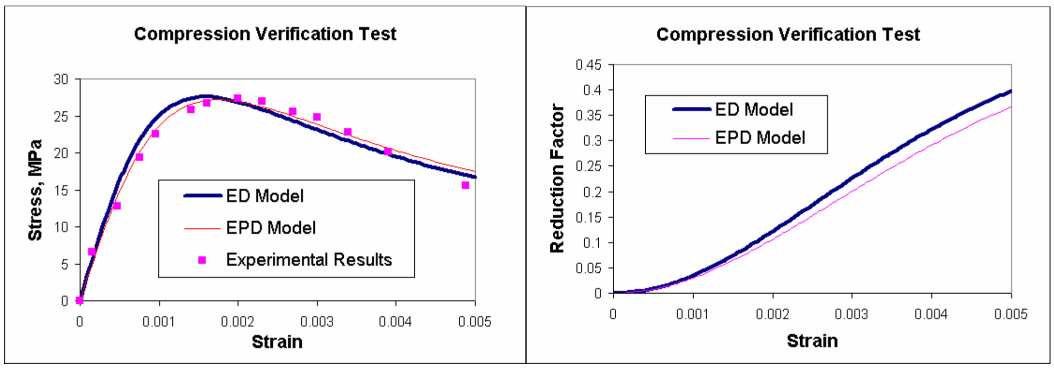
c) Relation between ε_{11} and χ^- d) Relation between φ^- and χ^-

Figure 4: Uniaxial compression verification results (EPD Model)



a) Tensile σ_{11} - ε_{11} curve b) Relation between ε_{11} and χ^{+}

Figure 5: Comparison of the tensile test results



a) Compressive σ_{11} - ε_{11} curve b) Relation between ε_{11} and χ^{-}

Figure 6: Comparison of the compressive test results

reduction factor is quite different for the two models. It is obvious from Figure (5b) that less reduction is needed in the case of the EPD model when compared to the ED model. The same trend of results is observed in the compression verification test, Figure (6b), although the magnitudes of the reduction factors in compression are less divergent from each other and many folds higher that those of the tensile test.

These results are justified by looking at Eq. (5), where the HFE function is written in terms of the combined damage variable Φ, the effective fourth-rank elasticity tensor $\bar{E}_{ijkl}$, and the elastic strain tensor ε_{ij}^{e}. In the ED model, there are no inelastic strains, and therefore, the entire strain increment is elastic, which gives a higher value of $\rho\psi^{e}$. In the EPD model, on the other hand, the strain tensor is additively decomposed into elastic and plastic components, thus reducing the magnitude of $\rho\psi^{e}$.

Another related outcome of the comparisons is the difference in effect of the hydrostatic stress on the tensile and compressive simulations in general. Although many mathematical forms of the reduction factor are possible, the form of χ chosen here to be identical for tension and compression, Eq. (6), clearly shows that the hydrostatic pressure effect is more dominant under compression than under tension; a result consistent with the literature of engineering mechanics.

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