REALIZATION OF FRACTIONAL ORDER PID CONTROLLER USING OPAMP CIRCUIT

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***ABSTRACT*-This paper concerned with realization of fractional order PID (FPID) by electronic circuit utilizing the syntheses of a fractance circuit that can be connected on operational amplifier either in the input or the output feedback of operational amplifier (OPAMP) circuit. Where the fractional order derivative (FD) and fractional order integral (FI) have been approximated to integer order rational transfer function depending on Carlson method. The continuous fractional expansion (CFE) and partial fraction expansion (PE) have been utilized on the (FI) and FD approximated transfer function to synthesis the fractance circuit. The analogue electronic circuit has been simulated using Circuit Wizard to realize the FD, FI and FPID circuits. The time domain response has been tested with sine wave input. The Bode plot for both real FD and FI** with that **the approximated one has been drawn.**

***Keyword:** Fractional order PID .FPID , Nintegerv.2.3Fractional control toolbox , Partial fraction expansion (PE) , Continuous fractional expansion (CFE), fractance*.

I. INTRODUCTION

Recently the fractional calculus become applied in PID controller which is called the fractional proportional, integral, derivative (FOPID) controller that include a fractional derivative and integral term, where each correspond to non-integer order either positive or negative power of Laplace operator s .It can be transformed to rational of two polynomial function in Laplace operator s in selected frequency band . This deduced rational transfer function has been synthesized to RC circuit using continues fractional expansion (CFE) or partial fractional (PF) expansion which called fractance circuit [1]. The fractance circuit can be utilized with OPAMP circuit either in feedback or input of it .The realization of FOPID taken into the count by many authors like Podlubny and I.Petras (2001) where the method of realization has been presented for fractional order system and synthesis of electronic circuit [2] .later the author B.T.Krishna and K.V.S.Reddy (2008) pointed to active and passive realization of fractional order of (1/2) [3]. The author G. Sridhar and K. Hrishikes (2012)[4] addressed design and to achieve dominant fractional by Simulink using MATLAB and give a comparison of results between the proposed with others in which they found the dominant fractional is the best .Then Lubomir Dorcak and Juraj Valsa (2013)[5] where pointed to analogue realization to FO dynamical system. In this paper the method of Carlosn has been used for the approximation of the FD and FI where on (CFE)and (PE) has been implemented in order to synthesis the fractance circuit for derivative and integration raised to non-integer order where used in structure of FPID controller. The electronic circuit realization done through circuit wizard simulation.

II. MATHEMATICAL EXPRESSION OF FRACTIONAL ORDER.

The mathematical representation for the (integration and differentiation) fractional given as:

$$D_a^{\alpha}u(t) = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} \quad u(t) \qquad if \ < 1 \\ \dfrac{d}{dt} \quad u(t) \qquad if \ \alpha = 0 \end{cases} \quad \dots (1)$$

It clear from the expression that for α negative it lead to fractional integration (FI) while if it is positive it lead to fractional differentiation (FD).

III. FRACTIONAL ORDER APPROXIMATION

In order to implement the derivative or integral term practically as analog electronic circuit utilizing synthesis method, the fractional order derivative or integration approximated to integer order rational transfer function

$$G(s) = \frac{Q^n(s)}{P^n(s)} \dots\dots (2)$$

Where $Q^n(s)$, $P^n(s)$ are polynomial of order n and the approximation take into account selected frequency band (wl:wh).The equivalent approximated transfer function G(s) can be synthesized as passive (RC) circuit using continues fractional expansion (CFE) which is called ladder form. The simplest procedure of fractance synthesis is based on the repeated division process of term Corresponding decomposition into the continued fraction in the most fundamental form[8][9] is:-

$$Y(s) = C_1 s + \cfrac{1}{R_1 + \cfrac{1}{C_2 s + \cfrac{1}{R_2 + \cfrac{1}{C_3 s + \cfrac{1}{R_3 + \cfrac{1}{C_4 s + \cfrac{1}{R_4}}}}}}} \quad \dots (3)$$

Where Ci and Ri should be rounded to obtain real valued components which called domino ladder circuit.

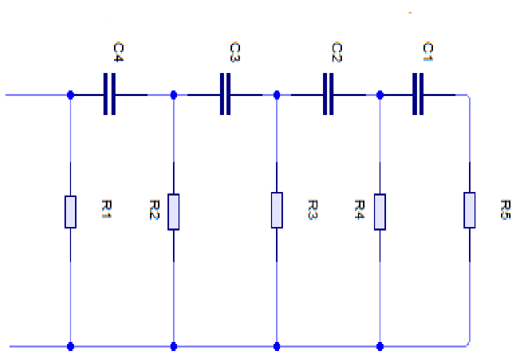


Fig (1) R, C domino ladder circuit

The second method is by using partial expansion (PE) for G(s) that will leads to cascaded parallel R and C circuits.

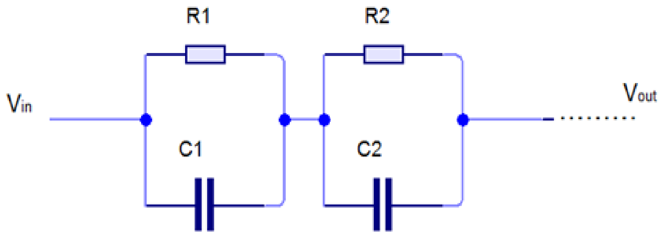
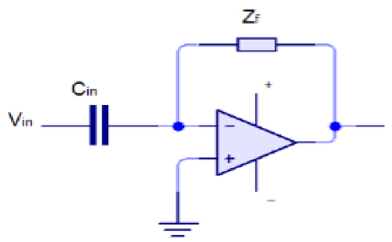


Fig (2) Cascaded parallel RC circuit

In order to realize FD controller by analog circuit using operational amplifier it is like the integer order circuit with additional circuit called fractance (Zf) which synthesized from G(s) by one of the methods CFE and PF as shown in Fig (3). While for the integration the synthesized Zf connected in feedback of OPAMP circuit with input resistance.



Fig(3) Analog FD circuit

IV. METHODS OF APPROXIMATION

Several method used to obtain the approximated rational fractional transfer function with integer order. These method given by Crone, Carlson, Matsuda, Ousta loop [6] where most usable, and each method has its own properties. Utilizing Matlab 2010 program and setting the special tool (Ninteger tool box) [7] to it by set path the following instructions in Matlab used to implement the approximation:

A. *Carlson method*

```
c = carlson(k, a, w, N);
minreal(c1)
```

k=gain, a=1/ fractional order, w=frequency, N=number approximate

B. *Crone method*

```
c = crone1(k, v, wl, wh, n);
 minreal(c)
```

k=gain, v= fractional order, wl =low frequency, wh =high frequency

V. FRACTANCE CIRCUIT

Design of fractance can be done easily using any of the aforementioned rational approximations and truncated CFE.Truncated CFE does not require any further transformation rational approximation. The Matlab instruction for CFE given as:

```
c = cfehigh(k, v, bandwidth, n)
minreal(c)
c1= cfelow(k, v, bandwidth, n);
minreal(c1);
```

k=gain,v=order,bandwidth=frequency,n= approximation number

The values of the electric elements, which are necessary for building a fractance, are then determined from the obtained finite continued fraction. If all coefficients of the obtained finite continued fraction is positive, then the fractance can be made of classical passive elements (resistors and capacitors). If some of the coefficients are negative, then the fractance can be made with the help of negative impedance converters [2]. It is in the same manner for realization of integrator just replacing the fractance in the feedback of the operational amplifier

VI .FRACTIONAL ORDER PROPORTIONAL, INTEGRAL, DIFFERENTIAL (FPID)

Collecting the terms FD and FI and proportional gain the fractional order Proportional, Integral, Differential (FPID) constructed as an improvement on classical PID. The mathematic expression as a function of time as follows:

$$y(t) = K_p x(t) + K_i D_t^{-\lambda} x(t) + K_d D_t^{\mu} x(t) \dots\dots\dots (4)$$

While the mathematic expression as a function of Laplace as follows:

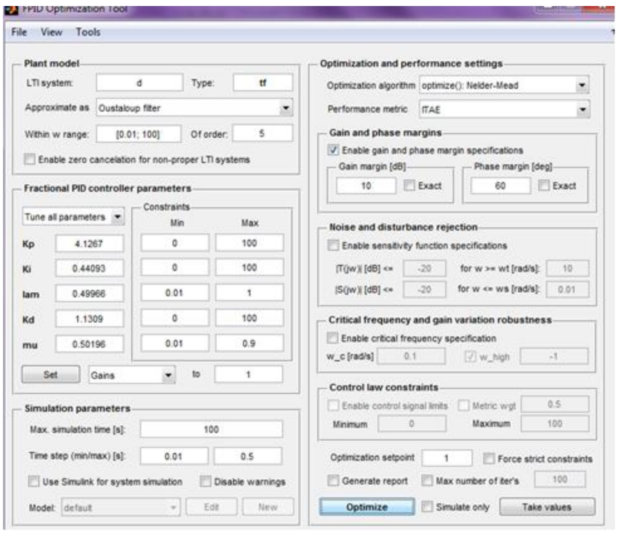
$$Y(s) = K_p X(s) + \frac{k_i}{s^{\lambda}} X(s) + K_d s^{\mu} X(s) \dots\dots\dots\dots (5)$$

VII. DESIGN OF FPID USING OPTIMIZATION.

From above equation (4) shows that controller contain five parameters (proportional coefficient, integral coefficient, differential coefficient, λ, μ) so to tune these five parameters requires optimization software for given plant transfer function with selected cost function and constraints. In order to get optimum value for these parameters that perform special specifications in time and frequency domain with such constraints leading to achieve the best desirable performance of the system and to adapt quickly to external disturbances by get rid of them or reduce their impact. Choosing the optimization tool in the FOMCON program which set to Matlab.

VIII. Application

FPID controller applied on the servo motors that has transfer function [10].
$G_p(s)$ = (3/10)/(s^2 +0.07s) .with unity feedback H=1. Performing the optimization on it as in GUI shown below the results are the parameters of FPID was:



Fig(4) Matlab optimization

$$c(s) = 4.1267 + \frac{0.44093}{s^{0.49966}} + 1.1309 s^{0.50196} \quad \dots (6)$$

The two non-integer order of Laplace operator s converted into two rational functions using Carlson method by:

```
s=fotf ('s')
H1=s^ (-0.49966)
ki= 0.44093;
a=2.0014;
w= [0.01:100];
N=5;
G2= carlson (ki,-a, w, N);
F= mineral (G2)
```

```
s=fotf ('s');
H1=s^0.50196
kd =1.0954;
a=1.1309;
w= [0.01:100];
N=5;
G= Carlson (kd, a, w, N);
A=mineral (G)
```

The results are for integration is:

$$= \frac{0.1579\, s^4 + 0.5712\, s^3 + 0.2009\, s^2 + 0.01346\, s + 0.0001449}{s^4 + 0.938\, s^3 + 0.1414\, s^2 + 0.00406\, s + 1.133e-005} \quad ..(7)$$

While for differentiation is**:**

$$= \frac{3.158\, s^{\wedge}4 + 2.963\, s^{\wedge}3 + 0.4466\, s^{\wedge}2 + 0.01282\, s + 3.58e-005}{s^4 + 3.618\, s^3 + 1.273\, s^2 + 0.08526\, s + 0.0009181} \quad \dots (8)$$

Performing CFE on the integration part instruction on the transfer function like:

```
f=tf((157.9*s^4+ 571.2*s^3+ 200.9*s^2+ 13.46*s +
0.1449)/(s^4+ 0.938*s^3+ 0.1414*s^2+ 0.00406*s +
1.133e-005))
[qv, expr, tex] = polycfe(f)
```

The results was as in the form of equation (3) with R,C values given in Table(1)

TABLE (1) R C values

C1	0.0023636 F	R1	159.9 Ω
C2	0.0058988 F	R2	820.0506 Ω
C3	0.010758 F	R3	1635.1844 Ω
C4	0.020634 F	R4	2938.4373 Ω
		R5	7237.4834Ω

The domino RC ladder circuit constructed for the values in Table (1) the active electronic circuit built as shown in Fig (3)

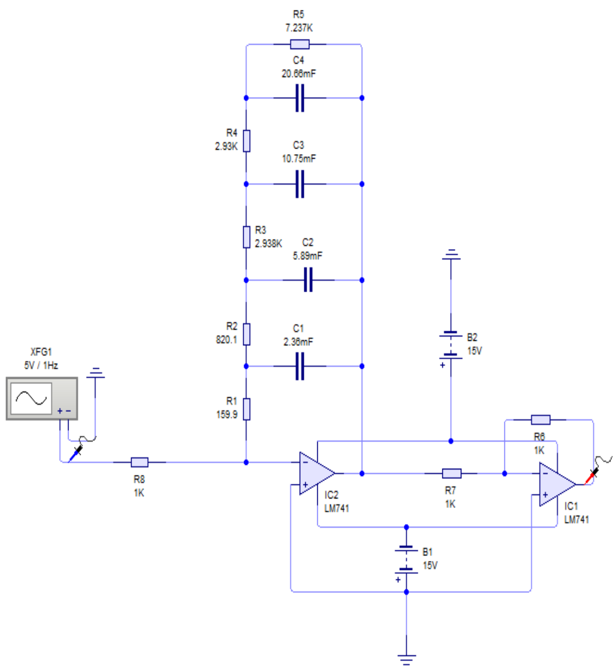
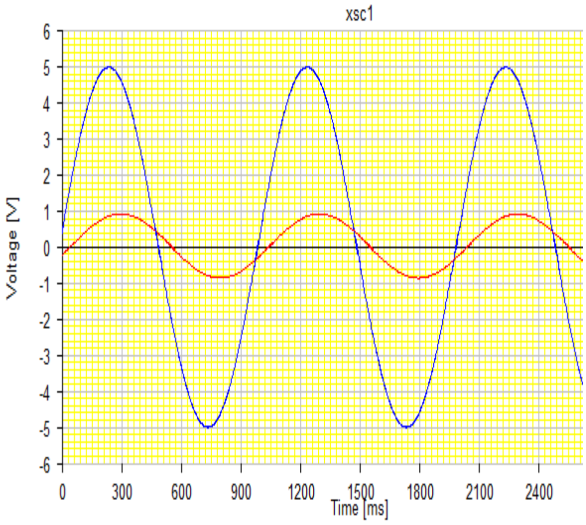


Fig (5) integration circuit

The test of the circuit shown for sinusoidal input in Fig (6) which has phase shift of α*90 degree.



.Fig (6) Test of the FI circuit with output red color, and input blue color

While the FD part has been realized using partial fractional expansion on the transfer function in 2 using Matlab instruction residue like:

b1=[3.059e6 2.87e6 432600 12420 34.68];

a1=[1 3.618 1.273 0.08526 0.000918 0];

[r,p,k]=residue(b1,a1)

The result given as:

$$\text{rpk}(s) = \frac{2.5047e6}{s+3.2323} + \frac{0.3023e6}{s+0.3017} + \frac{0.1286e6}{s+0.0707} + \frac{0.0855e6}{s+0.0133} + \frac{0.0378}{s} \ldots\ldots(9)$$

Which correspond to cascaded parallel RC circuit with impedance

$$z(s) = \frac{R}{RCs+1} \ldots(10)$$

When the matching between each term of the residue which correspond to impedance of parallel RC circuits then the values are given in Table (2).

TABLE (2)
R, C values from the resultant impedance

R1	0.775 MΩ	C1	0.399 µF
R2	1.002 MΩ	C2	3.307 µF
R3	1.82MΩ	C3	7.77 µF
R4	6.43 MΩ	C4	11.69µF
		C5	26.45µF

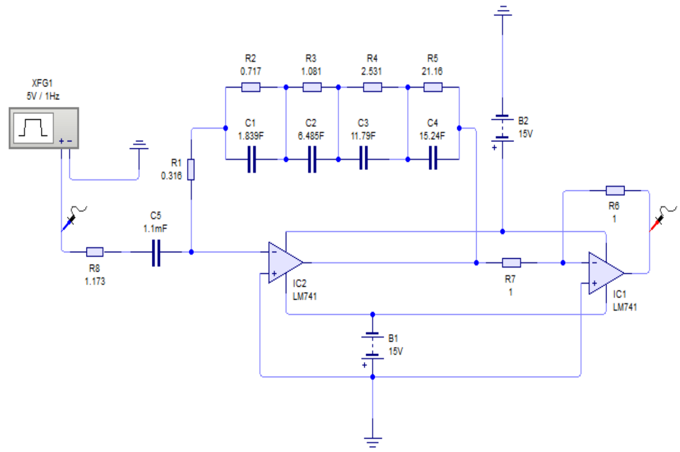
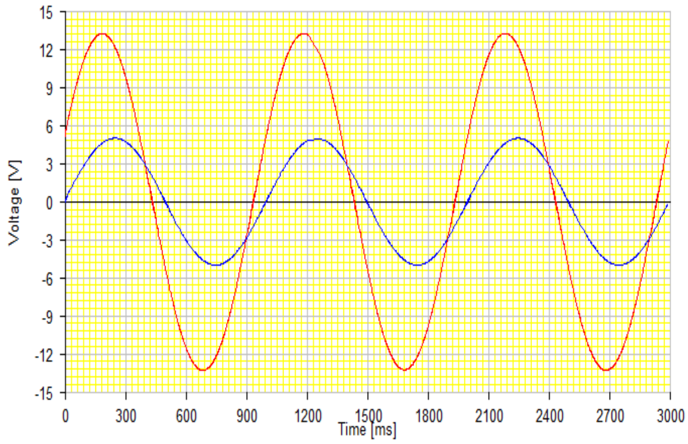


Fig (7) FD electronic circuit

When a sinusoidal signal applied to the circuit in Fig (7) the output is with phase shift about 0.501*90.



Fig(8)Test of FD output red color circuit with sinusiodal input blue color

The Bode plot for FI and FD for bothe the reail and approximated one shown in Fig(9 a-b)

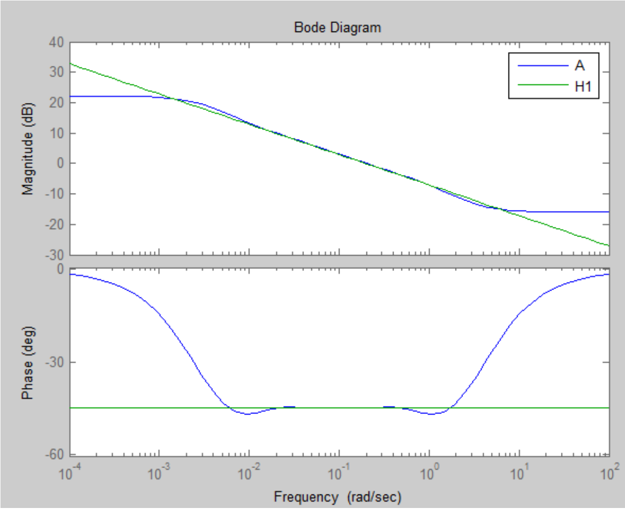


Fig (9-a) Bode plot of FI for the real (H1) and approximated transfer function (A),

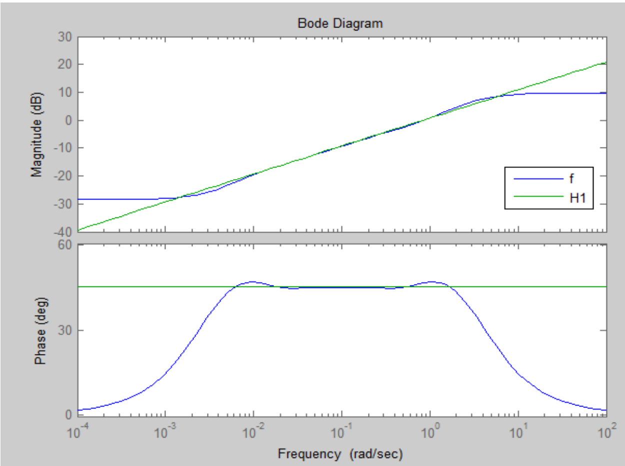
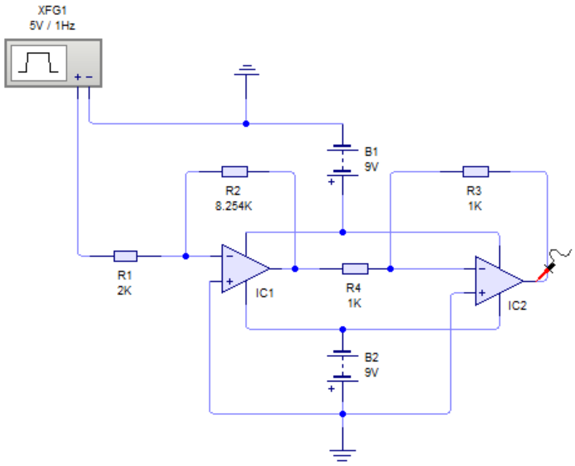


Fig (9-b) Bode plot of FD for the real (H1) and approximated transfer function (f)

The last step is realization of proportional control with gain k_p with inverting circuit given in the circuit which gives shown in Fig (10) :

$$k_p = \frac{R_2}{R_1} \ldots\ldots(11)$$



Fig(10) Proportional gain Kp circuit

Then collecting three circuits the for FPID controller electronic circuit where given in Fig (11) below using circuit wizard.

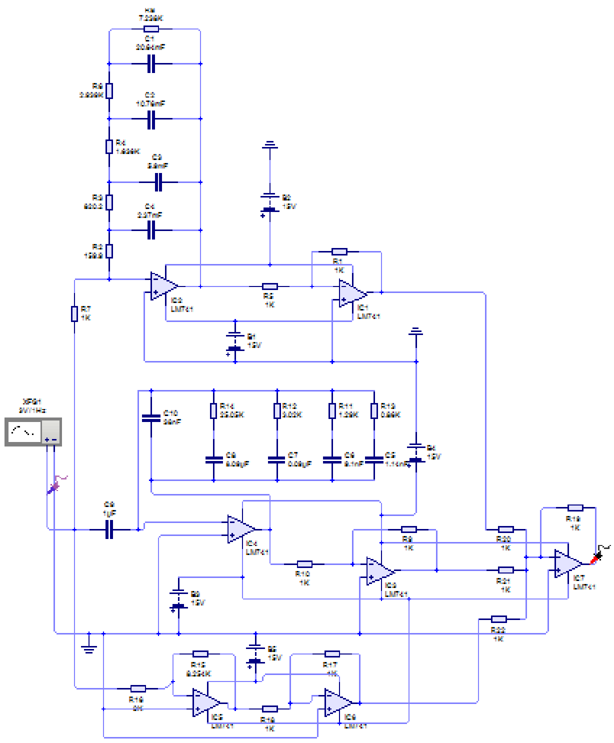


Fig (11) FOPID controller

IX. CONCLUSION

The fractional order differentiation or integration raised to positive or negative non integer order in Laplace operator (s) that result a change in both magnitude and phase in the Bode diagram. Fractional order derivative and integration has been approximated to rational transfer function for selected band width. Continues fractional expansion and partial fraction expansion can be used to synthesis the fractance as passive RC circuit in electrical circuit which has the form of ladder or cascaded circuits connected with OPAMP circuit to realize it as $FPI^{\lambda}D^{\alpha}$ controller. The synthesized circuit realized by electronic circuit simulating it on Wizard Circuit packages satisfy the results where a phase shift for sinusoidal inputs is fraction of ninety degree lead or lag.

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