Influence of specimen size and loading feature on fracture toughness of rocks

Abstract

Fracture toughness of mode I (opening mode) is an important parameter to describe the failure of rocks in many industries such as mining, tunneling, underground excavations, rock cutting and hydraulic fracturing.

This parameter was determined by two types of laboratory tests: three-point loading tests (SENB) and direct tension tests (DT) on notched specimens. This paper compares the toughness parameter of limestone measured from the two type of tests and from specimens of different diameters. The toughness parameter in DT tests was obtained by analysis of 27 specimens with different ratios of notch length to specimen diameter in finite element code Abaqus. The results of these analyses are then processed by code SPSS to derive a relation for the toughness parameter. To study the effect of specimen diameter, toughness parameter was measured from DT tests in four different diameter and then variation analysis was performed on the results. The rock elastic parameters were obtained from uniaxial compression tests.

An increase of about 20 percent in the measured toughness parameter was observed as the specimen diameter increases from 35 mm to 95 mm. Furthermore, the toughness parameters obtained from SENB and DT tests differ by nearly 16 percent.

1. Introduction

Fracture mechanic is a branch of solid mechanic which investigates the behavior of bodies with failure when exposed to stress and strain fields. One of the important branches of fracture mechanic is rock fracture mechanic. Fields, such as petroleum engineering, geological engineering, geotechnical engineering, and mining engineering, deal with rock fracture mechanics. Crushing rocks like blasting, sounding, rock cutting, tunneling, and hydraulic fracturing.

Growth mechanism of failure in rocks is of brittle fracture type. Brittle fracture which refers to rapid and unsustainable growth of failure in rocks often occurs in hard materials with negligible plasticity. Therefore, concepts of linear elastic fracture mechanics (LEFM) are applied for them.[1,2,3]

To state stress around the failure tip considering the infinity of its value, stress intensity factors are used; critical value of stress intensity factor in one loading mode is called fracture toughness which is a mechanical feature of materials. This parameter is considered as an inherent feature of materials which describes time, location, and reason of fracture occurrence in a material and is a parameter of stress intensity factor (SIF) k. Three basic loading modes for a failure include opening mode (mode I), sliding mode (mode II), and tearing mode (mode III); critical stress intensity factor corresponding to three basic modes of failure surface displacement are determined by K_{IC}, K_{IIC}, and K_{IIIC}, respectively [4].

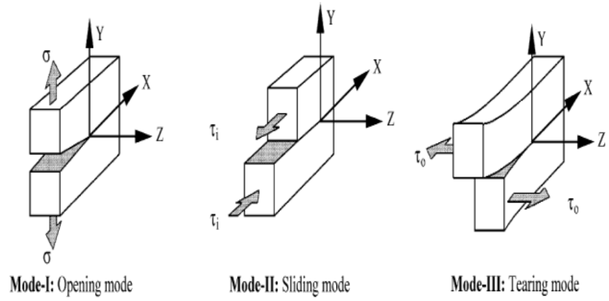

Fig. 1 Basic fracture modes

Fracture toughness of rocks is measured using a wide spectrum of laboratory methods and specimens. However, the fracture toughness obtained from these methods has not been equal for the same rock type.

Factors such as specimen size, non-isotropic nature of rock, size of fracture process region near the failure tip (size of plasticity region of failure tip), micro- and macro-structure of rock, rock storage conditions (e.g. humidity), and so on are involved in this process.

According to these ambiguities, in this paper, first, effect of diameter on fracture toughness values of mode I of limestone was examined using a number of tests on cylindrical under direct tension loading (DT) rock specimens with four different diameters. Then, several tests were conducted on single edge notched beam (SENB) and DT specimens and the results were compared in order to determine effects of loading type. Moreover, to obtain a relation for fracture toughness of DT model, a number of dimensionless models were modeled in finite element code ABAQUS[5,6,7].

2. Introducing crack tip parameters

Knowing K for every special problem, displacement and stress field around failure tip can be determined. General form of relation for stress intensity factor is as follows in Eq. (1) [8], [9]:

$$K = f(g)\sigma\sqrt{\pi a} \qquad (Pa\sqrt{m}) \tag{1}$$

where f(g) is a parameter which depends on specimen and crack geometry and loading conditions. For pure mode I, stress field is stated as follows [8], [9]:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{2}$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \tag{3}$$

$$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \tag{4}$$

3. Explaining laboratory specimens:

3-1: In SENB test, a cubic specimen with standard dimensions was used. In this test, the specimen was placed on plain bearing and then under a three-point bending load (Fig. 2). Failure position in this specimen can be asymmetrical to the load application points. Thus, while applying three-point loads, combined loading conditions occurred. With changing the crack position, modes I and II can be made [10]:

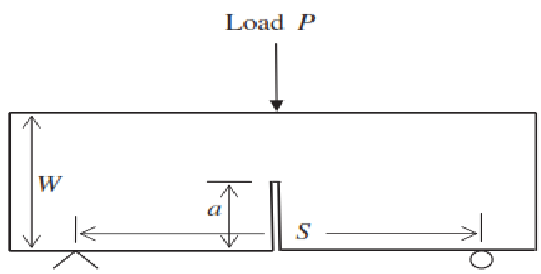

Fig. 2 Geometry and loading of the cubic specimen for SENB test [11]

Fracture toughness in SENB test was calculated using the following relation:

$$K_{1Q} = f(\frac{a}{W})\frac{P_Q.S}{B.W^{1.5}} \quad (5)$$

Where P_Q is maximum load, S is loading span, B is specimen depth, and W is specimen height.[11]

In Relation (30.2), $f(\frac{a}{W})$ is the dimensionless parameter calculated as follows:

$$f(\frac{a}{W}) = 3\sqrt{\frac{a}{W}}[1.99 - \frac{(a/W)(1-(a/W))(2.15-3.95(a/W)+2.7(a/W)^2)}{2(1+2(a/W))(1-(a/W))^{3/2}}] \quad (6)$$

where a/W ratio is equal to specimen height to crack length ratio (a/W=0.5).

3-2: To perform DT tests on cylindrical specimens, four joints with diameters of 35, 55, 75, and 95 mm were used. The specimen had L/D ratio of 3 and notch width of 1 mm.

The tests were done as follows: after preparing the specimens, to get reliability about flatness of both specimen endings, the device was adjusted on the loading rate of 0.002 mm/s and loading was started. The output related to load value and displacement of moving joint was recorded by a data-logger. Also, the maximum load causing the specimen fracture was considered as fracture load.

Fig. 3 Fracture and growth of failure in DT specimen

4. Numerical modeling

As can be observed in Eq. 1, to determine stress intensity factor, f(g) should be calculated. To calculate this factor, finite element code ABAQUS was used. In Fig. 4, the finite element model made in this software is demonstrated. In this model, 4-node elements were used; also, singular elements were used around failure tip.

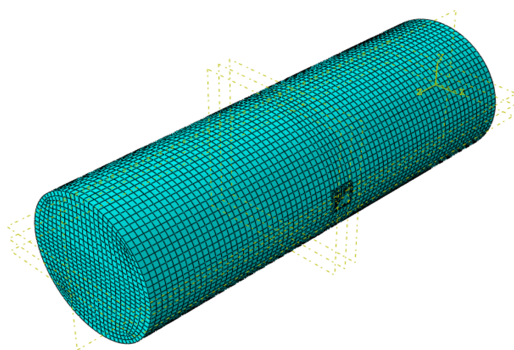

Fig. 4 Finite element model for DT specimen.

Geometrical dimensions of the part was precisely according to the one explained in the section of laboratory specimen. Young's modulus and Poisson ratio of the used material were 9GPa and 0.25 respectively. The form factors were determined as follows: first, values of stress intensity factors were calculated using the fracture load obtained from the test. Then, by inserting the required geometrical values, stress intensity factors, and load calculated in Eq. 1, f(g) values were calculated. These values are given in Table 1.

$D_{(mm)}$	$K_{(MPa.m\char`^0.5)}$	$P_{(KN)}$	f(g)
35	0.824	1.355	2.498
55	0.766	2.46	2.517
75	0.849	4.35	2.511
95	1.023	7.44	2.523

Table 1. f(g) values for DT specimens.

Also, to obtain a relation for the form factor of DT model, 27 models were made in finite element code ABAQUS in addition to the above modeling. In this section, to determine the form factor, load 2N was assumed and also specimens with diameters of 35, 55, 75, and 95 mm and L/D and a/D ratios of 2.5, 3, 3.5 and 0.4, 0.5, 0.6 were used. By investigating the results obtained from the software, it was observed that form factor values for DT model did not depend on L/D ratio and diameter and were only related to a/D ratio changes. Finally, input results in SPSS statistical analysis software are presented in Table 2 .

Sample Name	P(N)	a/D	f(g)
DT1	2	0.4	2.533
DT2	2	0.5	3.602
DT3	2	0.6	5.505

Table 2. Input results in SPSS

5. Discussing the results

5-1: To determine fracture toughness using SENB method, it is sufficient to place the P load obtained from the test resulting in the part fracture in Eq. 5 to calculate fracture toughness values. In Table 3, the results obtained from the test and fracture toughness values are demonstrated.

Sample Name	P_{Cr}(kn)	Y(a/W)	K($MPa.m^{0.5}$)
Senb1	0.87	3.902	1.215
senb2	0.95	3.902	1.323

Table 3.Fracture toughness values for SENB specimens

In this table, P_{cr} is forced which is imposed at fracture moment. By averaging K_I values in pure mode I, fracture toughness value in mode I (K_{IC}) will be 1.269 $MPa.m^{0.5}$.

To determine fracture toughness values using DT method, fracture load values obtained from the laboratory in the four mentioned diameters were considered the ABAQUS software input. Finally, fracture toughness value in mode I (K_{IC}) was presented for DT specimen as in Fig. 5 .

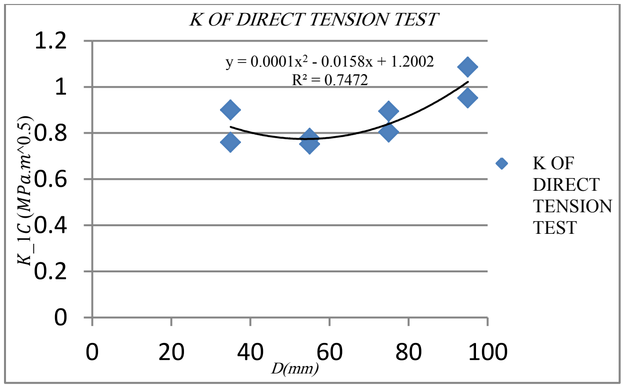

Fig. 5 varation of fracture toughness by variation of diameter in DT specimen.

In Fig. 5, fracture toughness value was generally increased with increasing diameter so that, with increasing diameter from 35 to 95 mm, K_{IC} value was increased by about 20%; however, at smaller diameters, a relatively uniform trend was followed. Therefore, this method can be used to obtain rock fracture toughness at small diameters.

In Fig. 6, K_{IC} results obtained from two DT and SENB methods were compared with each other. Accordingly, the fracture toughness of mode I obtained from SENB method was almost 16% larger than that by DT method. According to the above results, fracture toughness value of mode I for a type of limestone relatively depended on the specimen geometry and loading conditions. This difference can be caused by failure development and growth in the specimens.

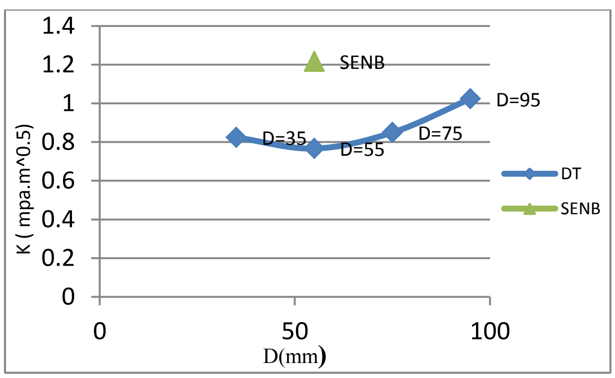

Fig. 6 K_{1c} results obtained from two DT and SENB methods.

5-2: To analyze regression of fracture toughness values of mode I for DT specimens and obtain a relation for the form factor of this geometry, SPSS[12] statistical analysis software was used. To select the best fitness, a function was used which had maximum coefficient of determination. According to the obtained results from this software, all the functions had a significant value of 0, indicating that the regression was significant. Also, maximum value of R-square belonged to quadratic and cubic functions with the coefficient of determination of 1. Finally, quadratic function was selected as the most suitable fitness. In Fig.7, fitness diagram for different default functions of SPSS software is presented.

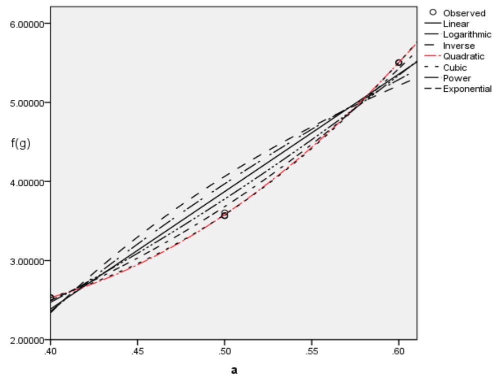

Fig. 7 Fitness diagram of different functions in SPSS software

Finally, the relation of dimensionless form factor for the cylindrical specimen under DT load with edge crack becomes as Relation (7):

$$f(g) = 7.099 - 28.929(a/D) + 43.783(a/D)^2 \quad (7)$$

where (a/D) ratio is equal to notch length to specimen diameter ratio.

5-3: One-way Analysis of Variance on the results of fracture toughness of mode I for different diameters of DT specimen was done in SPSS software to evaluate hidden effects of diameter factor on K_{IC} value. In Table 4, value of significant factor was more than the defined significant value (0.05); i.e. diameter changes did not influence fracture toughness values of mode I.

K_{IC}	***Sum of Squares***	***df***	***Mean Square***	***F***	***Sig.***
Between Groups	*0.0704*	*3*	*0.023*	*4.04*	*0.101*
Within Groups	*0.0234*	*4*	*0.006*		
Total	*0.0945*	*7*			

Table 4. One-way Analysis of Variance on the results of fracture toughness of mode I for different diameters of DT.

6. Conclusion

1- The new test of cylindrical specimen with edge crack and under DT load was presented; the advantage of this method was in simplicity of testing and preparation of parts.

2- Fracture toughness of mode I obtained from DT method had 16% difference from the values obtained from SENB method.

3- In DT specimens, according to Fig 5. with increasing diameter, fracture toughness value of mode I was generally increased, but according to One-way Analysis of Variance which was done on the results of fracture toughness of mode I for different diameters of DT specimen, diameter changes didn't influence fracture toughness values of mode I.

4- According to the regression analysis which was done on the results of numerical modeling, Relation (7) for the form factor of DT specimen were presented:

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