D-Q model and control of a three - phase induction motor considering mutual flux saturation effect

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***Abstract*— The field oriented control of the three-phase induction motor (IM) has become an increasingly popular control methodology, where control of the flux linkage can be effectively decoupled from the torque, allowing for segregated flux and speed controls of the IM. However, if flux saturation effect was ignored, reductions in the reliability of the control of the IM and degradation in performance are expected. In this paper, the MATLAB SIMULINK d-q model of the three - phase IM is analyzed, and a d-q control schematic of the IM is applied. Results of torque and speed responses to the reference inputs were obtained in two separate cases; the first one supposes that the stator current is linearly proportional to stator voltage at all times (no flux saturation exists), and the second case, which is more practical, involves the mutual flux saturation. A comparison between the two cases in terms of torque and speed performances of the IM is yielded.**

Keywords—Field oriented control; Saturation; Induction motor;

List of Symbols

V_s , $V_r^{'}$: Stator and rotor voltages.

i_s , $i_r^{'}$: Stator and rotor currents.

L_s , $L_r^{'}$: Stator and rotor inductances.

L_m : Magnetizing inductance.

R_s , $R_r^{'}$: Stator and rotor windings' resistances.

ψ_s , $\psi_r^{'}$: Flux linkages of the stator and rotor.

ψ_m : Magnetizing flux linkage.

T_e : Electromagnetic Torque.

w, w_r : input mechanical reference and rotor electrical angular velocities.

θ : Rotor position.

T_r : Rotor time constant.

$\Delta\psi_m$: Approximate polynomial function to describe the saturation curve.

ϕ_s , ϕ_r : Stator and rotor fluxes.

p : number of pole pairs.

I. INTRODUCTION

The three phase induction motor is renowned for its robustness and relatively low cost. However, it was not applicable before controlling its speed and torque characteristics, due to the fact that it offered a fixed-speed limitation. Various techniques emerged to solve this problem; primarily scalar control and field oriented control. The FOC has been first introduced by Blaschke and K.Hasse who first proposed the vector control approaches for induction motors [1][2]. FOC introduces a prime advantage over scalar control that it can effectively decouple the control of flux linkage apart from the control of the output torque. Moreover, one can employ proportionate integral derivative (PID) controllers in the FOC control schematic although AC quantities are evident; this is because of the fact that the rotating d-q coordinate plane is synchronously rotating at the same speed of a desired AC quantity, turning its behavior to be like a DC quantity; hence a PID controller can be introduced. Obtaining a valid mathematical model for the IM is a key factor towards completing the FOC scheme. Generally speaking, there are many models that reflect the motor behavior in arbitrary reference frame. It is however useful to change the frame into a synchronously rotating reference frame that rotates at the same speed of one of the AC quantities (stator and rotor voltages, currents, and flux linkages, as well as the air gap flux linkage and the magnetizing current) in order to simplify equations and conduct FOC.

Depicting saturation effect and including it in the control scheme greatly enhances the accuracy of simulations. Since saturation effect is essentially related to the magnetizing inductance (Xm), choosing a d-q model that implies either the magnetizing current or the air gap flux linkage as state space variables suffices to estimate flux saturation effect. In such condition, one has to solve the set of mathematical differential equations depicting the model of the IM. Numerical methods help greatly in this case, especially the finite element analysis method [3]. However, one could incorporate the effect of saturation if other models are implemented, by re-modifying the control scheme blocks. Changing the mathematical model is justified because some state space variables can not be measured or otherwise not easily accessed. Another distinct approach relies on continuously updating the values of the motor's parameters, especially the magnetizing inductance. Ideally these parameters are fixed, but because of the flux saturation that takes place, the respective values slightly deviate and change. If one could analyze this change of values and consecutively update the parameters used in the IM mathematical d-q model, then the saturation effect is included, without the need to add further control blocks or assigning specific state space variables likewise. Many research articles focused on identifying and updating the machine's parameters as the motor operates, stating at the same time that this change is evident primarily because of the saturation effect (in case of the magnetizing inductance) and temperature deviations (for the other parameters, especially the rotor resistance) [4, 5]. Other researches focused on the effect of parameters' deviation on the FOC scheme [6]. Representing the magnetizing inductance by a decaying function with the respect to the magnetizing current (and consequently expressing it through a power function) yields the inclusion of the saturation effect [7]. Quite similarly, one can consider the saturation effect by updating the value of stator inductance itself, by relating it to a power function that contains the fixed unsaturated value of the stator inductance [8]. Relating the rotor flux to the magnetizing current through a non-linear function also proved to incorporate saturation effect [9].

Magnetic flux saturation is divided into the saturation of the mutual flux between the stator and rotor of the IM and the saturation of the leakage fluxes of the stator and rotor. Although leakage flux saturation is depicted through researches [10], the main and effective flux saturation is primarily coming from the mutual flux between the stator and the rotor of the IM, hence the leakage flux saturation is ignored in many cases, including the work of this paper. All previous modeling analysis and the methods to include the saturation effect can be discretized in order to be compatible with microprocessor – base controllers. Discretization methods are followed in various researches in order to obtain the corresponding models with the saturation effect modeled as well [11]. This paper introduces the FOC of three phase induction motor, represented by q-d model, with implication of saturation effects. The saturation is considered simply by inserting typical curve values to reflect the actual relationship between voltage and current, or between flux and current of the motor. It has been done using the hybrid continuous / discrete modified control scheme blocks provided by the SIMULINK software platform. Simulation results have demonstrated how the motor react with the controller which proved the suitability of FOC for controlling saturated induction motors. The paper is organized as follows. Section II presents the mathematical model of the IM. Simulation results and their analysis are depicted in section III. Finally, the conclusions and future recommendations are given in section IV.

II. INDUCTION MOTOR MODEL AND FIELD ORIENTATION

The mathematical model for the IM that has been applied is depicted in equations 1-5:

$$V_{qs} = R_s i_{qs} + \frac{d\psi_{qs}}{dt} + w\psi_{ds} \quad (1)$$

$$V_{ds} = R_s i_{ds} + \frac{d\psi_{ds}}{dt} - w\psi_{qs} \quad (2)$$

$$V_{qr}' = R_r' i_{qr}' + \frac{d\psi_{qr}'}{dt} + (w - w_r)\psi_{dr}' \quad (3)$$

$$V_{dr}' = R_r' i_{dr}' + \frac{d\psi_{dr}'}{dt} - (w - w_r)\psi_{qr}' \quad (4)$$

$$T_e = 1.5\frac{d(\psi_{ds} i_{qs} - \psi_{qs} i_{ds})}{dt} \quad (5)$$

The previous direct and quadrature flux linkages of the stator and rotor are accounted for by employing the machine's inductances as follows:

$$\psi_{qs} = L_s i_{qs} + L_m i_{qr}' \quad (6)$$

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr}' \quad (7)$$

$$\psi_{qr}' = L_r' i_{qr}' + L_m i_{qs} \quad (8)$$

$$\psi_{dr}' = L_r' i_{dr}' + L_m i_{ds} \quad (9)$$

Where L_s, L_r' consist of the leakage and magnetizing inductances of both the stator and rotor, as equations 10, 11 show:

$$L_s = L_{ls} + L_m \quad (10)$$

$$L_r' = L_{lr}' + L_m \quad (11)$$

The FOC of the schematic depends primarily on finding the rotor position θ that is defined by:

$$\theta = \int (w_r + w_m)dt \quad (12)$$

The importance of finding θ is to successfully obtain the d-q components of the measured three phase currents. This is achieved through the transformation equations 13, 14, where i_a, i_b, i_c represent the measured (abc) currents:

$$i_{ds} = \frac{2}{3}[\frac{i_b}{2}(1.7\sin\theta - \cos\theta) + \frac{i_c}{2}(-\cos\theta - 1.7\sin\theta)] \quad (13)$$

$$i_{qs} = \frac{2}{3}[\frac{i_b}{2}(\sin\theta + 1.7\cos\theta) + \frac{i_c}{2}(\cos\theta - 1.7\sin\theta)]$$
(14)

Now the corresponding flux linkage of the rotor can be found by applying equation 15, which depicts the rotor flux linkage in the S domain:

$$\psi_r' = \frac{L_m i_{ds}}{1 + T_r S} \quad (15)$$

Where T_r represents the rotor time constant, defined by:

$$T_r = \frac{L_r'}{R_r'} \quad (16)$$

The essence of the FOC depends on setting a desired torque T_e^* and a desired rotor flux ϕ_r^* for the IM, where the desired d-q stator currents i_{ds}^*, i_{qs}^* can be accordingly evaluated through equations 17, 18:

$$i_{qs}^* = \frac{2}{3}\frac{1}{p}\frac{L_r'}{L_m}\frac{T_e^*}{\psi_r'} \quad (17)$$

$$i_{ds}^* = \frac{\psi_r'^*}{L_m} \quad (18)$$

Where $\psi_r'^*$ is obtained when ϕ_r^* is set to a PI controller that receives ψ_r' as a feedback quantity.

Since the desired stator currents are now known, one can apply the inverse Park, Clarke transformations in order to obtain the desired (abc) currents, as equations 19-21 show:

$$i_a^* = -i_{qs}^* \sin\theta + i_{ds}^* \cos\theta \quad (19)$$

$$i_b^* = \frac{i_{ds}^*}{2}(1.7\sin\theta - \cos\theta) + \frac{i_{qs}^*}{2}(1.7\cos\theta + \sin\theta)$$
(20)

$$i_c^* = -i_a^* - i_b^* \quad (21)$$

Now one can subtract the actual (abc) stator currents from the desired (abc) stator currents in order to yield the error. This difference is sent to the pulse width modulation (PWM) block that is responsible for generating the triggering pulses to the gates of the three-phase inverter supplying the IM, resulting in evident change of the supply voltage if there has been a difference between the desired and actual currents.

In most cases, the beneficiary of the IM is not interested in assigning desired flux and desired electro-magnetic torque, but rather in assigning desired rotor mechanical speed. The control schematic must imply additional blocks to interpret the desired mechanical speed into the desired flux and electro-magnetic torque. A look-up table embedding the relationship between the mechanical speed of the rotor and the rotor flux is applied in order to obtain the desired rotor flux. As for the desired electro-magnetic torque, one can apply the desired speed into a specific transfer function that acts as a PI controller, where the resultant value is the desired electro-magnetic torque. Figure 1 depicts the control schematic responsible for operating the IM per the previous equations and conditions.

This control scheme for the moment is ignoring the effects of core saturation, thus is relatively impractical. The saturation effect can be included by modifying the SIMULINK blocks of the IM mathematical model. Nevertheless, the external control scheme blocks are not modified; because they are not affected nor they affect the saturation phenomenon that takes place within the IM itself. As for the internal blocks (the IM mathematical model), the modified quantities are the d-q components of the magnetizing flux linkage, where these two values shall be updated in a manner that depicts the saturation curve instead of implying a linear relationship.

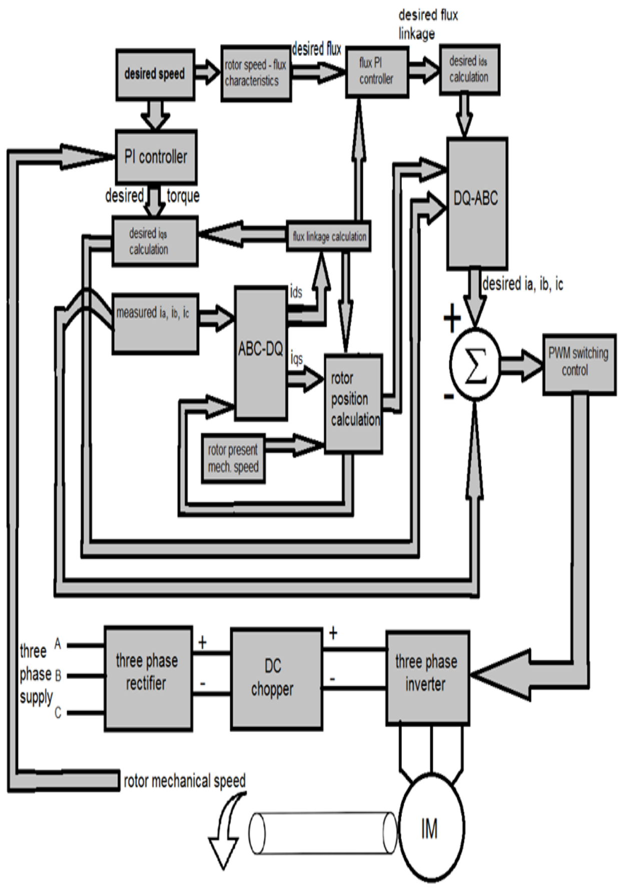

Figure 1: The implemented FOC scheme of the IM.

In order to account for saturation effect, define K as:

$$K = \frac{L_m}{L_{ls}} + \frac{L_m}{L'_{lr}} \tag{22}$$

The updated values of the d-q components of the magnetizing flux linkage can be obtained when a proper polynomial function for saturation curve approximation is involved in the equations determining the new values for the magnetizing flux linkage components. Such approximation polynomials are obtained numerically or are already built-in in the SIMULINK platform. Equations 23, 24 illustrate the update process for the magnetizing flux linkage components necessary for depicting the saturation effect in the IM:

$$\psi_{mq}^{sat} = \psi_{mq} - [K(\psi_{mq} \Delta\psi_m)] \tag{23}$$

$$\psi_{md}^{sat} = \psi_{md} - [K(\psi_{md} \Delta\psi_m)] \tag{24}$$

III. SIMULATION RESULTS

The previous control scheme was implemented on the SIMULINK platform; where two cases have been tested: the first case assumes that no mutual flux saturation is evident; in other words, the polynomial function that approximates the saturation curve is always linear. The other case assumes a saturated curve, in which the mutual flux saturation is hopefully depicted. In both cases, the input is the same (the same desired speed and desired electromagnetic torque) in order to establish a comparison between the two cases. The IM under simulation testing is a 200 hp, 460 volts, 60 Hz, two pole-paired IM. Figures 2, 3 illustrate the desired speed and electromagnetic torque, where the x-axis represents the time in seconds.

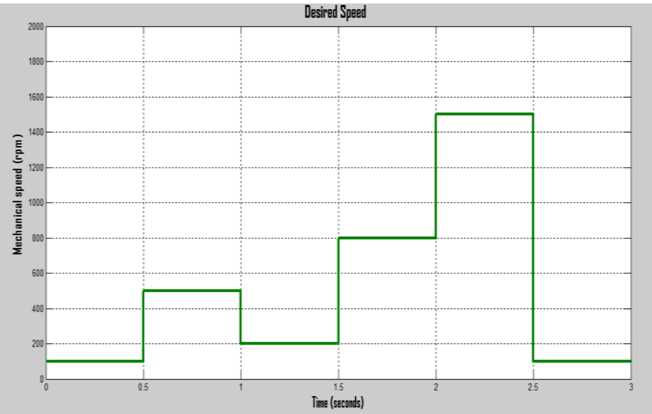

Figure 2: Randomly chosen desired speed.

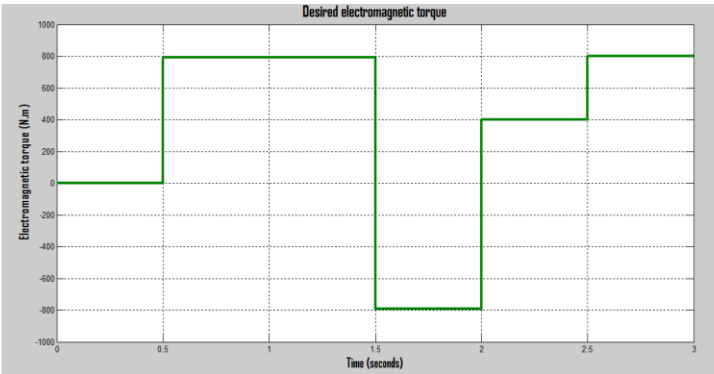

Figure 3: Randomly chosen desired electromagnetic Torque.

Figure 4 represents the current in stator winding (a) where the ideal case is assumed, whereas figure 5 illustrates the same current when the mutual flux saturation is accounted for.

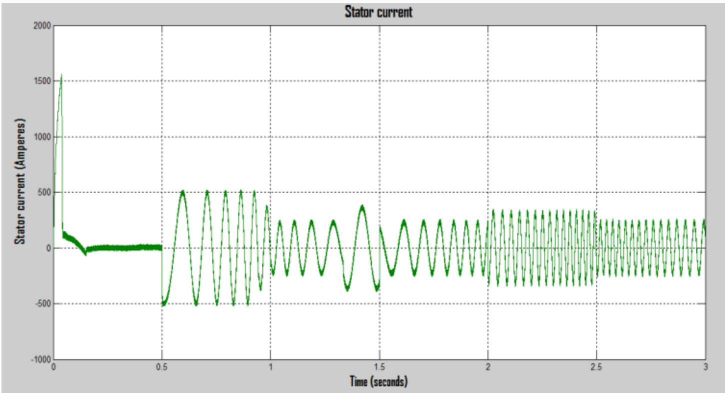

Figure 4: Stator winding (a) current assuming no saturation effect.

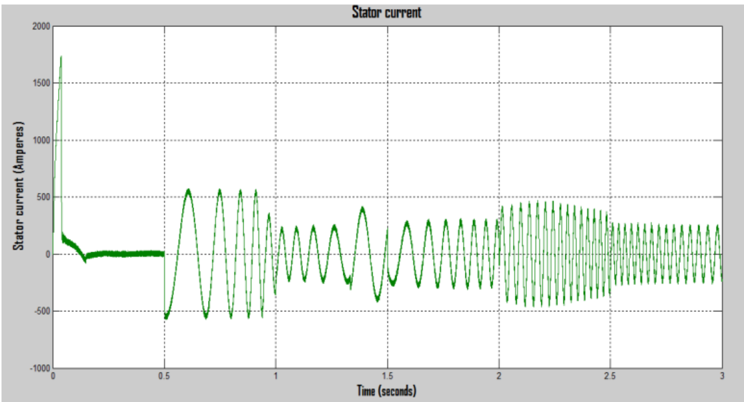

Figure 5: Stator winding (a) current assuming saturation effect exists.

Figures 4, 5 clearly show that the mutual flux saturation caused the current in stator windings to increase. This side effect could be tolerated in the steady –state operation, however, it is harmful in the transient and sub-transient regions. The starting current (without saturation effect) is nearly 1500 A, whereas in the practical case (with saturation effect) the starting current increased by approximately 250 A.

Figures 6, 7 depict the mechanical speed of the shaft without accounting for saturation effect and with it, respectively. This speed must be in accordance with the desired speed, however - as it is obvious in the two figures - the transition from one desired speed to another does not happen instantly; certain delay time is evident before stabilizing into the new desired speed.

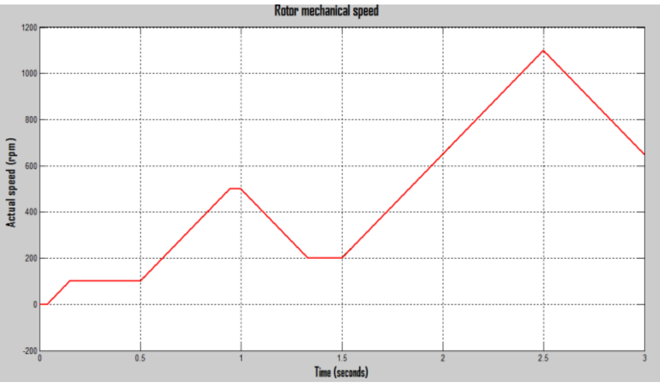

Figure 6: Rotor mechanical speed assuming no saturation effect.

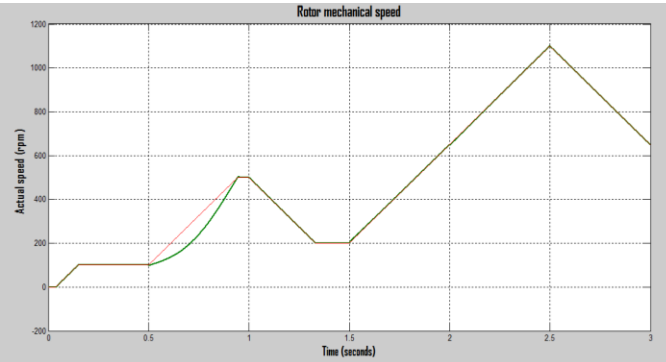

Figure 7: Rotor mechanical speed assuming saturation effect exists.

Obviously from figures 6, 7, mutual flux saturation made the transition between one desired speed and the other exponential with time instead from linear. However, this behavior is not persistent; some transition periods are not affected by saturation effect.

The resultant electromagnetic torque is depicted in figures 8, 9, without and with the flux saturation effect, respectively. It is noticed that the transition in electromagnetic torque also requires sufficient time to achieve the desired value. It is noticed that flux saturation effect causes performance disturbances and torque deviations from the desired values. Although this effect is not persistent throughout the periods, the saturation effect on the developed electromagnetic torque has a relatively greater impact than on the mechanical speed of the shaft. As for the DC voltage coming from the DC chopper to the inverter, it was noticed that the flux saturation effect did not impose any considerable changes, even in the sub-transient region. Figures 10, 11 illustrate these findings.

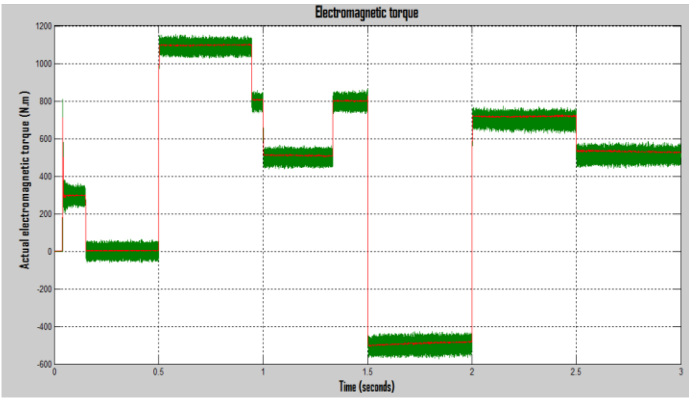

Figure 8: Electromagnetic torque assuming no saturation effect.

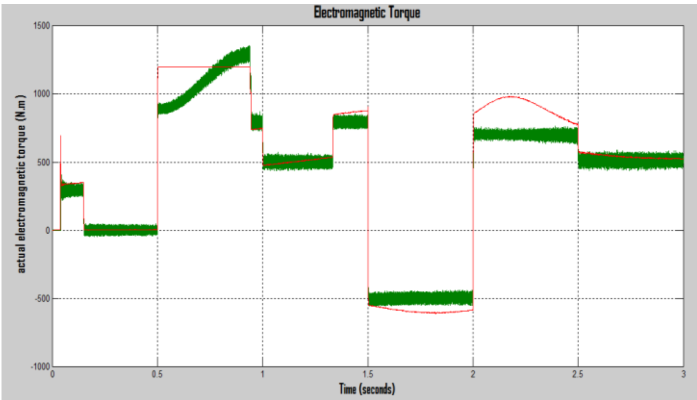

Figure 9: Electromagnetic torque assuming saturation effect exists.

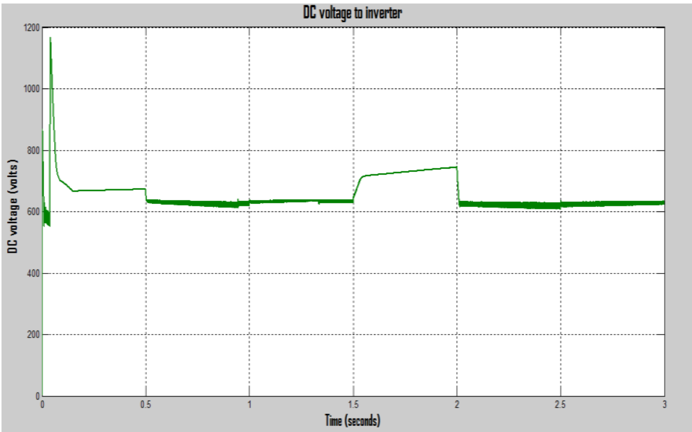

Figure 10: DC voltage to inverter assuming no saturation effect.

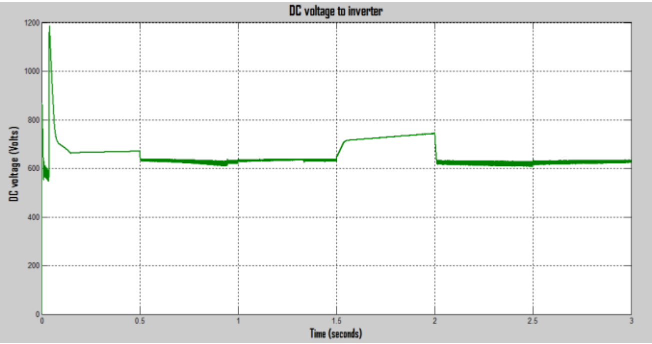

Figure 11: DC voltage to inverter assuming saturation effect exists.

IV. Conclusion

The SIMULINK platform for field oriented control of the three - phase IM has been analyzed and implemented, where two different cases were introduced: the first case implies a non practical situation where no mutual flux saturation takes place. In this case, the relationship between the stator voltage and stator current is assumed linear. In the second case, the mutual flux saturation is depicted through an approximate polynomial function that updates the values of the magnetizing flux linkage components, and consecutively accounting for the effect of saturation. The second case is more practical and showed higher starting stator current, exponential speed transitions, and electromagnetic torque deviations. For future implementation, Another FOC scheme with different methodology depicting the saturation effect can be constructed, where the results of the two saturation cases can be compared for validation.

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