DIRECT-CONTACT HEAT EXCHANGER WITH PHASE CHANGE: THEORETICAL EXPRESSION FOR HEAT TRANSFER COEFFICIENT AND VOLUMETRIC HEAT TRANSFER COEFFICIENT OF A TWO-PHASE BUBBLE

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ABSTRACT
The present investigation deals with experimental and theoretical phenomenological study of three-phase direct-contact heat exchanger, for n-pentane-water system. The test section consisted of a cylindrical perspex column 17.2cm i.D. and 1m long, in which, distilled water, was to be confined. Liquid n-pentane drops were injected into the hot water filled column, through a special design of two distributors (A) and (B) at the bottom of the column. The liquid n-pentane drops rose on their way up and evaporated into two-phase bubbles at atmospheric pressure.

A study of speed and high resolution camera films taken during the heat transfer process rendered information regarding the bubble shape, bubble size, and evaporation rates of n-pentane drops evaporating in hot water.

The study was devoted to express the effect of process variables on heat transfer coefficient, and volumetric heat transfer coefficient and effectiveness, the experimental work was designed for this purpose in the following experimental ranges:
1. Inlet temperature of water \(T_{ci}\) (38.1-46.1) °C.
2. Water volumetric flow rate \((Q_w)\) (9.8-49.03) cm³/s.
3. n-pentane volumetric flow rate \((Q_d)\) (0.9636-1.927) cm³/s.

using two special designs of distributors (A) and (B). A theoretical analysis of evaporation of the multidrops moving in a direct-contact heat exchanger, uses the radial and tangential velocity components, assuming spherical drops in a test column, the irrotational and inviscid flow (laminar flow condition) was between two concentric spheres. A theoretical model for the prediction of heat transfer coefficient for water \((h_w)\), and volumetric heat transfer coefficient \((U_v)\) was derived and compared with the experimental data, as follows:

\[
Nu_w = 0.9213x \sqrt{\left( \frac{\frac{1}{2}}{\frac{3}{2}} \right) \left( \frac{\Delta T_{liq-a}}{\Delta T_{liq-d}} \right) Pe} \left( \frac{k_d \rho_d C_p_d}{k_c \rho_c C_p_c} \right) \frac{1}{\sqrt{U_v}}
\]

and

\[
U_v = 4\pi R_d^2 N_d h_c
\]

This Equation gives mean deviation of 7.79% with \(T_{ci}\), 17.5% with \(Q_w\), and 10.859% with \(Q_d\) for distributor (A), while it gave 9.08% with \(T_{ci}\), 12.73% with \(Q_w\), and 8.96% with \(Q_d\) for distributor (B).

INTRODUCTION
The study of direct-contact heat transfer to evaporating drops (dispersed phase) rising due to buoyancy in another liquid (continuous phase) is of interest, because the heat transfer rates in direct-contact evaporators are usually much higher than those of conventional surface-type heat exchangers because of the large heat transfer area for a given volume [Dammel and Beer (2000)].

Direct-contact between two fluids associated with the phase change of one of the fluids has shown many advantages. Due to the higher effective heat transfer coefficient, relative simplicity of design, and absence of the scaling surfaces associated with direct-contact evaporators and condensers. Applications include geothermal heat recovery, sea water desalination, waste heat recovery, energy storage systems, and Production of steam generation for the Rankine power cycle from the direct-contact vaporization of water with lead-bismuth eutectic in Pb-Bi/ water reactor (PBWR) [Battya et al. (1982), Battya et al. (1983), Seetharamu and Battya (1989), Buongiorno et al. (1999), Dammel and Beer (2000)].

Battya et al. (1982, 1983) investigated the theoretical analysis of direct-contact latent heat between immiscible liquids in a counter flow spray column, using kerosene-water and n-pentane-water systems. Smith et al. (1982) developed an analytical model for the evaluation of the volumetric heat transfer coefficient in direct-contact evaporation between two immiscible liquids in a spray column. The analysis was divided into a pregglomerative and a postagglomerative stages. They also conducted experiments with cyclopentane as the dispersed phase and stagnant water as the continuous phase,
using two distributor plates.

Tadrist et al. (1987) studied the experimental and theoretical aspects of the vaporization of a multi droplet by direct-contact of refrigerant R113 and n-pentane dispersed into a column of water flowing counter currently. They developed an analytical model giving the evolution of void fraction and the temperature of each fluid along the exchanger column and also estimated the volumetric heat transfer coefficient and the exchanger efficiency.

Jacobs and Golafshani (1989) described the characteristics of a spray column type direct-contact liquid-liquid heat exchanger, and developed a steady-state one-dimensional multiphase flow model, using isobutane-brine system. In the model they assumed the heat transfer to be controlled by conduction within the drops.

Mori (1991) studied the evaporation of drops of a volatile liquid sprayed upward in an immiscible liquid flowing down in a vertical column, and derived an expression for the volumetric heat transfer coefficient in a counter flow spray column. The expression of the volumetric heat transfer coefficient was used to predict its values under some particular column operating conditions, which were then compared with relevant experimental data found in the literature.

Rasheed (1999) studied the direct-contact evaporation of a drop moving in a stagnant column of an immiscible liquids, using n-pentane-water, 2-methyl pentane-water, and n-pentane/2-methyl pentane-water systems. A theoretical analysis of evaporating droplets in an immiscible liquid was developed by solving the governing equations of the motion and heat transfer numerically by Runge-Kutta method, assuming a spherical drop in a column of stagnant immiscible liquid at uniform temperature.

**EXPERIMENTAL SECTION**

The experimental system is shown schematically in Fig.(1). The test section consists of a cylindrical perspex column of 17.2cm inside diameter and 1m long, in which the test fluid, water, was to be confined, and a cylindrical perspex-made water jacket, served as constant-temperature water bath and also contributed to minimizing the heat losses from the test column by circulating water around the test column. The water was supplied at the top of the test column through the distributor at the center of the test column and flowed downward to exit through the bottom by a constant-temperature circulating bath equipped with a thermostatically electric controlled heater. The optimum water height in the column was 85cm. The temperature of inlet water is measured by calibrated sheath copper-constantan thermocouple(1), inserted at the top of the inlet water line, and the temperature of outlet water is measured by calibrated sheath copper-constantan thermocouple(2), inserted below distributor section. The n-pentane entered the test column at constant temperature of 30°C from the bottom through a distributor and flowed upward in droplet form by a pump. The temperature of the n-pentane is measured by calibrated sheath copper-constantan thermocouple(3), which was inserted before the distributor. The n-pentane vapor is collected at the top of the test column, and condensed in a vertical condenser by circulating cooling ethanol/water mixture. The condensate is collected in a storage tank. The condensate was analyzed by using gas chromatography to be reused in the experiments.

Two types of distributors were used, distributor (A) of 25 orifices, each orifices of 1 mm diameter, and distributor (B) of 15 orifices, each orifices of 2 mm diameter, the two type was made of teflon with an aluminum plate of 5 mm thickness. They were arranged in a square pitch.

To analyze the shape and size of the bubble was photographed by a video camera at 30 frames/s. The bubble size was recorded by a computer.

**DATA ANALYSIS**

Data were acquired by direct measurement, enlarged consecutive pictures were developed from the speed and high resolution camera films. For these measurements the entire “drop” as well as the vapor phase were taken as ellipsoidal shape. The instantaneous equivalent spherical diameter of the two-phase bubble was calculated using the measured horizontal and vertical diameters, using a steel ball to calibrate the horizontal and vertical diameters from the following equation [Simpson et al. (1974), Raina et al.(1984)], and its values were calibrated with camera.
\[ d_D = \left[ D_0^3 + \left( \frac{M-1}{M} \right) \right]^{1/3} \]  

(1)

By assuming no heat losses from the test column, and knowing inlet and outlet temperatures of water and n-pentane, one calculates the logarithmic mean temperature difference, heat transfer rate, and volumetric heat transfer coefficient from the following heat balance Eq.(2), with neglecting superheating and \( F=1 \):

\[ q = \dot{m}_l C_p (T_{ci} - T_{co}) = \dot{m}_d V + \dot{m}_l C_p \Delta T_{\text{LM}} F \]  

(2)

The heat transfer coefficient for continuous phase (water) was estimated from the two following equations, Eq.(3), and Eq.(4).

\[ h_c = \frac{U V}{4 \pi \cdot R^2 D N_d} \]  

(3)

and

\[ N_d = \frac{\phi (\%)}{100} \frac{4}{3 \pi} R^3 D \]  

(4)

**MATHEMATICAL MODEL**

Consider the system of drop (\( d_l \)) (as shown in Fig.(2)) and its nucleating bubble (\( d_v \)) to be held stationary in a steady uniform flow of an immiscible non volatile liquid (c). Assume that the spherical bubble which has a diameter (2a) was nucleated mainly heterogeneously inside drop (\( d_l \)). This vapor bubble starts to grow in size due to the addition of molecules of vapor emitted from the inner surface of the drop.

The dispersed liquid drop (\( d_l \)) of diameter (2b) is situated within the continuous immiscible liquid (c). The transfer of thermal energy from liquid (c) to drop (\( d_l \)) is due to the temperature difference between (c) and (\( d_l \)). This energy is balanced by the release of the latent heat of evaporation from drop (\( d_l \)) leading to the nucleation of bubble (\( d_v \)).

Cooling caused by the evaporation of drop (\( d_l \)) produces the following temperature scales (see Fig.(2)).

\( \Delta T_{\text{dl}} \) represents the amount of superheat of the system.

\( T_{dl} = (T_{dvl} + T_{dvl})/2 \) represents the saturation temperature of the volatile drop (\( d_v \)) corresponding to the local operating pressure of the system.

\( \Delta T_{\text{dl}} \) represents the degree of thermodynamic non-equilibrium of the system.

Assuming that the surface tension at the two interfaces (that is, bubble (\( d_v \))- drop (\( d_l \)) and drop (\( d_l \))-liquid (c)) is large enough to provide for keeping the spherical shape of both the bubble and the drop throughout the period of evaporation.

The irrotational and inviscid flow between two concentric spheres is given by [Lamb (1945)] and utilized by [Kendoush (2001)] to calculate the drag forces of bubble swarms. The radial and tangential velocity components of this flow are given, respectively, by the following equations:

\[ V_r = \frac{U \cos \theta}{(1-x)} \left[ x - \left( \frac{b}{r} \right)^3 \right] \]  

(5)

and

\[ V_\theta = \frac{U \sin \theta}{(1-x)} \left[ x + \frac{1}{2} \left( \frac{b}{r} \right)^3 \right] \]  

(6)

where :

\[ x = \left( \frac{a}{b} \right)^3 \]

The overall Nusselt number is given by [Baird and Hamielec (1962)]:

\[ N_{\text{u,dl}} Pe_{\text{el}}^{1/2} = \left[ \frac{2 \pi}{\phi U_o} \left( V_\theta \right)_{x=b} \sin \theta \cdot d \theta \right]^{1/2} \]  

(7)

Substituting Eq.(6) into Eq.(7) we get the following :

\[ N_{\text{u,dl}} = 0.9213 \left[ Pe_{\text{el}} \left( x + \frac{1}{2} \right) \left( 1-x \right) \right]^{1/2} \]  

(8)

The total transfer rate is given as follows:

\[ q = h_{\text{dl}} \left( 4 \pi - a^2 \right) \Delta T_{\text{dl}} \Delta T_{\text{dl-v}} = h_c \left( 4 \pi - b^2 \right) \Delta T_{\text{c-dl}} \]  

(9)

where : \( \Delta T_{\text{dl-v}} = T_{\text{dl}} - T_{\text{dv}} \) and \( \Delta T_{\text{c-dl}} = T_c - T_{\text{dl}} \), hence :

\[ h_{\text{dl}} = h_c \frac{\Delta T_{\text{c-dl}}}{x/3 \Delta T_{\text{dl-dv}}} \]  

(10)

Substituting Eq.(10) into Eq.(8) and rearranging, we get :

\[ N_{\text{uc}} = 0.9213 \left[ \frac{x + \frac{1}{2}}{1-x} \right]^{1/2} \left( \frac{\Delta T_{\text{dl-dv}}}{\Delta T_{\text{c-dl}}} \right) Pe_{\text{el}}^{1/2} \left[ \sum_{l=1}^{n} \left( \frac{k_{el} \rho_{el} \phi_{el}}{k_c \rho_c C_p c} \right)^{1/2} \right]^{1/2} \]  

(11)

where :

\[ N_{\text{uc}} = \frac{k_l (2b)}{k_c} \]  

(12)

and

\[ Pe_{\text{el}} = \frac{U (2b)}{\alpha_c} \]  

(13)
The merits of Eq. (11) are the following:

1. It shows the inverse relationship between the rate of heat transfer and the superheat (that is, $\Delta T_{c-d}$). This fact has been confirmed experimentally by various authors [e.g. Sideman and Taitel (1964), and Shimaoka and Mori (1990) among others].

2. It gives a decrease in the rate of heat transfer with respect to the vaporization ratio $x$, therefore it calculates the instantaneous heat transfer coefficient. Since $x = 1 - \frac{m}{m_o}$, Eq. (11) becomes as follows:

   $$Nu_c = 1.07 \left( \frac{3 \ m_o}{m} + \frac{m}{m_o} - \frac{5}{2} \right)^{\frac{1}{2}}$$

   (14)

   for $k_d / k_c = 0.2$, $\rho_d / \rho_c = 0.59$, $Cp_d / Cp_c = 0.6$, $Pe_x = 19.1$ and $\Delta T_{c-d} = \Delta T_{d-c}$.

3. It gives the dependence of the rate of heat transfer on the coolant properties through the group $\left( k_d \rho_d Cp_d / k_c \rho_c Cp_c \right)^{\frac{1}{2}}$. This dependence was appreciated by Oguz and Sadral (1987) in their numerical solution. The presence of the property group in Eq. (11) provides a method for the selection of the best volatile coolant for any process.

RESULTS AND DISCUSSION

1. Qualitative Observations

   The experimental study and visual observations, made by the present investigation have revealed that the nucleated volatile dispersed liquid drops begin to grow with vapor accumulating on the top of the residual liquid, during its rise through the continuous immiscible liquid medium. The vapor volume increases with time acting like a balloon to lift the unevaporated dispersed liquid through the continuous phase medium.

   The evaporating dispersed liquid drops have been visualized as a two-phase bubbles, which starts as spherical drops, change to ellipsoids and finally to cap-shaped bubbles in agreement with [Sideman and Taitel (1964), Simpson et al. (1974), and Raina et al. (1984)].

2. Comparison of Heat Transfer Coefficient and Volumetric Heat Transfer Coefficient Equations with the Experimental Data

   The present experimental results of heat transfer coefficient and volumetric heat transfer coefficient for n-pentane-water system are compared with the present theoretical equations, Eq.(3) and Eq.(11), for both distributors (A) and (B):

   Fig.(3) and Fig.(4) illustrate the comparison of the experimental data for n-pentane-water system with the predictions of the present theoretical equation given by Eq.(3) and Eq.(11), for distributor (A).

Fig.(2): The geometry of the problem.
Fig. (5) and Fig. (6) indicate the effect of inlet temperature of water on heat transfer coefficient and volumetric heat transfer coefficient. Volumetric heat transfer coefficient decreases with increasing inlet temperature of water, due to the increase in the logarithmic mean temperature difference (driving force), and increase in the overall resistance of heat transfer, this is also reported by [Sideman and Taitel (1964), Sideman et al. (1965), and Sideman and Gat (1966)].

The results shown in Fig. (7), and Fig. (8) clearly indicate two operating ranges. The first range, heat transfer coefficient and volumetric heat transfer coefficient decreases with increasing water volumetric flow rate because of increasing the rate of coalescence compared with the rate of break-up in this range, and the second range, heat transfer coefficient and volumetric heat transfer coefficient increases with increasing water volumetric flow rate because of forming a smaller droplets which have less tendency to coalesce and high tendency to break-up of two-phase bubbles. Increasing in break-up of two-phase bubbles will increase the effective heat transfer area per unit volume of the heat exchanger, and increase the volumetric heat transfer coefficient, this is reported by [Sideman et al. (1965)].

The effect of n-pentane volumetric flow rate on heat transfer coefficient and volumetric heat transfer coefficient has been demonstrated in the following two Fig. (9), and Fig. (10). Heat transfer coefficient and volumetric heat transfer coefficient increases with increasing n-pentane flow rate for two distributors (A) and (B). This increase is attributed to the fact that in higher n-pentane flow rate, smaller bubbles are formed. These small bubbles have large interfacial areas leading to high volumetric heat transfer coefficient, this is reported by [Sideman et al. (1965) and Shahidi and Özbelge (1995)].
It is clear from Figs.(5) to (10) that the predicted theoretical values of heat transfer coefficient and volumetric heat transfer coefficient, agree more closely with experimental values, with Mean deviation 7.79%, 17.5%, and 10.859% for distributor (A), and 9.08%, 12.73%, and 8.96% for distributor (B). The slight deviation between experimental and theoretical values of heat transfer coefficient, and volumetric heat transfer coefficient, for both distributors (A) and (B) is attributed to the drastic change in the density of the two-phase bubble.
CONCLUSIONS
Parametric analysis of direct-contact heat exchanger in a counter-current column of immiscible liquid has been carried out. The predicted results compared favourably with the present experimental results. From the analysis the following conclusions are made:
1. The shape of the two-phase bubbles observed in the present study was changing from nearly spherical to ellipsoidal and sometimes to spherical-cap.
2. The theoretical equation of heat transfer coefficient and volumetric heat transfer coefficient for continuous phase (water) immiscible with n-pentane droplets is derived as follows:

\[ Nu_t = 0.9213 \left( \frac{x + \sqrt{x^2 + 1}}{2} \right)^{1/2} \left( \frac{\Delta T_d - \Delta T_v}{\Delta T_c - \Delta T_d} \right)^{1/2} Pe^{1/2} \left( \frac{k_d P_d C_d}{k_c P_c C_c} \right)^{1/2} \]

The above equation was found to give satisfactory comparison with the experimental results.
3. The volumetric heat transfer coefficient values fall with an increase in the inlet temperature of water.
4. Small-diameter nozzles associated with faster nozzle velocities, and smaller droplets, yield higher volumetric heat transfer coefficient, and larger heat transfer coefficients, i.e. [distributor (A) gave higher values of volumetric heat transfer coefficient than distributor (B)].

NOMENCLATURE
a bubble radius, m
b drop radius, m
\( C_p \) specific heat at constant pressure, J/kg.°C
d\( o \) droplet diameter of the two-phase bubble, m
dh horizontal diameter of the bubble, m
dv vertical diameter of the bubble, m
D\( o \) initial drop diameter, m
F correction factor
h heat transfer coefficient, kW/m².°C
k thermal conductivity, W/m.°C
m unevaporated mass of liquid in the drop, kg
m\( o \) initial mass of liquid of the drop, kg
\( \dot{m} \) mass flow rate, kg/s
M density ratio of liquid density to vapor density of n-pentane
N\( d \) number of droplets per unit volume, 1/m³
Nu overall nusselt number (h d\( o \)/k)
Pe Peclet number (U d\( o \)/a)
q heat transfer rate, kW
Q volumetric flow rate, cm³/s
r polar coordinate, m
R\( d \) droplet radius of the two-phase bubble, m
T temperature, °C

U uniform velocity of the flow, m/s
U\( v \) volumetric heat transfer coefficient, kW/m³.°C
V optimum volume of heat exchanger, m³
V\( r \) radial component of fluid velocity, m/s
V\( t \) tangential component of fluid velocity, m/s
X vaporization ratio

GREEK LETTERS
\( \lambda \) latent heat of vaporization, J/kg
\( \phi \% \) n-pentane holdup percentage
\( \theta \) polar coodinates, degree
\( \Delta T \) temperature difference, °C
\( \Delta T_m \) logarithmic mean temperature difference, °C
\( \rho \) density, kg/m³

SUBSCRIPTS
C continuous phase
d dispersed phase
i inlet condition
o outlet condition
V vapor

REFERENCES
Buongiorno, J., N.E. Todreas and M.S. Kazimi.1999. Thermal design of a lead-bismuth cooled fast reactor with in-vessel direct-contact steam generation. to be Published.


